COMPARATIVE ANALYSIS OF RECEIVER POSITION COMPUTATIONAL ALGORITHMS

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BACHELOR OF ENGINEERING

IN

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CERTIFICATE

This is to certify that the project report entitled **"COMPARATIVE ANALYSIS OF RECEIVER POSITION COMPUTATIONAL ALGORITHMS"** that is being submitted by Sri Haritha Deevi (160115735078) and Deepak Raya Velgapuni (160115735091) in partial fulfillment for the award of the **Bachelor of Engineering** in **Electronics and Communication Engineering** from **Chaitanya Bharathi Institute of Technology,** Hyderabad during the academic year 2018-2019 as a record of bonafide work carried out by them under my supervision.

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DECLARATION

This is to certify that the work reported on the present project entitled **COMPARATIVE ANALYSIS OF RECEIVER POSITION COMPUTATIONAL ALGORITHMS** is a record of work done by us in the Department of Electronics and Communication Engineering, Chaitanya Bharathi Institute of Technology, Osmania University. This report is based on the project work done entirely by us, it doesn't involve plagiarism and any information procured from resources has been mentioned in the references.

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ABSTRACT

Indian Regional Navigation Satellite system (IRNSS) is going to be an independent, indigenous navigation satellite system fully controlled by India, planned by ISRO. A system was designed of regional navigation satellite constellation, as an alternate to GPS constellation, for providing space based navigation support to various land, sea and air navigation users over the Indian region. The continuous visibility of GEO and GSO satellites for near-equator regions provides a promising alternative for regional navigation.

Accuracy of receiver position is an important parameter; even the small errors could lead to large position error. Navigation system is an over-dimensioned and over-determined system, for more than four satellites in the view of receiver. An estimation problem with so much redundant and yet noisy data is solved using various algorithms.

The main aim of this project is to perform comparative analysis of receiver position computation algorithms, namely least mean squares, Bancroft method and recursive least squares algorithm by considering computation time, complexity, accuracy parameters. This gives a scale for section of appropriate and efficient usage of algorithms based on application.

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CHAPTER 1

INTRODUCTION

1.1 AIM

To perform comparative analysis of GNSS receiver position computational algorithms.

1.2 OBJECTIVE

To study and compare the following position estimation algorithms:

1. Least mean squares algorithm.
2. Bancroft method.
3. Recursive least squares algorithm.

1.3 LITERATURE SURVEY

The project requires analyzing RINEX files that are being received from the GNSS receiver and extracting the data required for the computation of parameters using LMS, RLS and Bancroft algorithms on the MATLAB software.

Solving for position involves finding the roots of nonlinear equations either by using numerical methods or through multivariate quadratic equations. Few of the algorithms which perform the above kind of solution are least mean squares, recursive least squares and Bancroft method.

Least mean squares is a stochastic algorithm while recursive least squares is deterministic.The Bancroft method allows obtaining a direct solution of the receiver position and the clock offset, without requesting any "a priori" knowledge for the receiver location.

1.4 MOTIVATION

Accuracy and time constraints play an important role in satellite navigation systems. There are many proposed algorithms that give the position estimate from satellite navigation data. The choice of algorithm based on application plays an important role to the economy of navigation oriented industries. In order to make a better choice, the users need to have a prior knowledge about different algorithms that give a position solution. Having a comprehensive comparative analysis gives a helping hand to such type of problems. LMS and RLS algorithms have wide applications in signal processing.

1.5 ORGANISATION OF THE REPORT

The chapters in this report will give an overview of GPS and IRNSS satellite navigation systems, mathematical and qualitative description of LMS, Bancroft and RLS algorithms.

The organization of report is as follows:

Chapter 1: Introduction to project thesis.

Chapter 2: Introduction to GPS, its architecture, working principle and errors.

Chapter 3: IRNSS – Its background, development, architecture, signals, services and navigation data formats.

Chapter 4: Qualitative and mathematical description of position estimation algorithms.

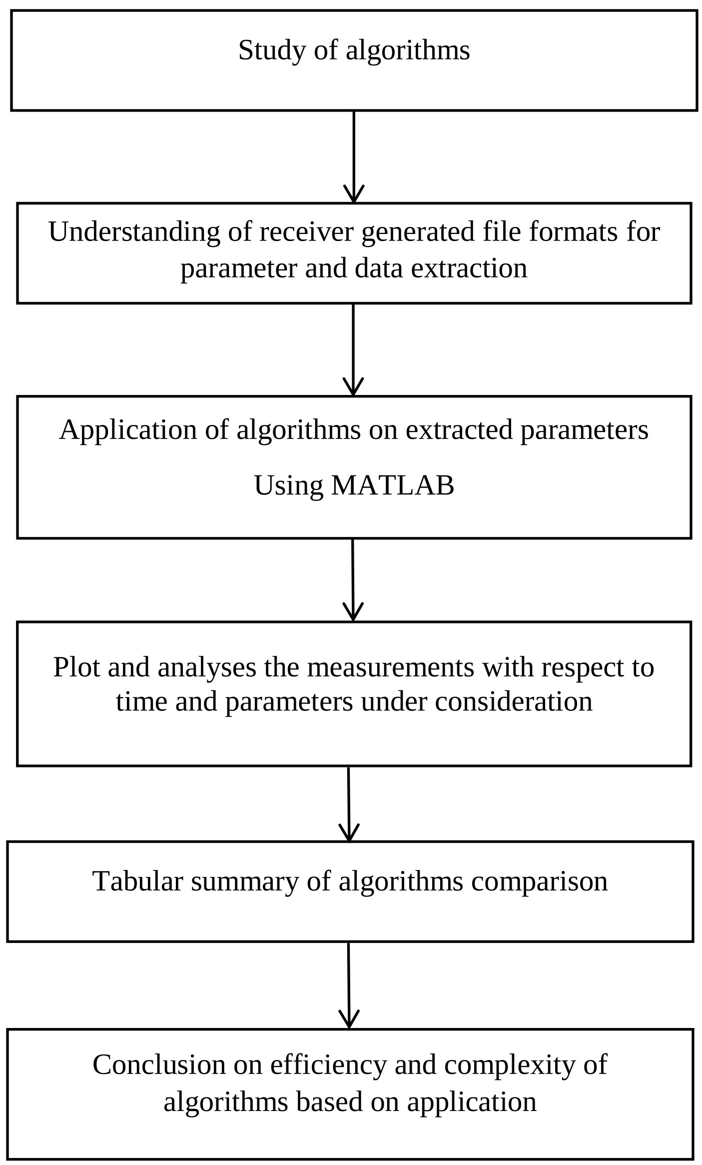
Chapter 5: Implementation of position estimation algorithms explained using flow charts.

Chapter 6: Analysis and discussion of results.

Chapter 7: Conclusion of the project and future scope.

1.6 METHODOLOGY

LMS, RLS and Bancroft algorithms are implemented by using the MATLAB software. **MATLAB** (**mat**rix **lab**oratory) is a multi-paradigm numerical environment developed by MathWorks. Data extracted from navigation data files is fed to the code to obtain user position using different algorithms and is compared based on the computational parameters. The flow of the project is as follows:



CHAPTER 2

GLOBAL POSITIONING SYSTEM

2.1 INTRODUCTION

The Global Positioning System was introduced in 1960 under the U.S. Air Force. The first satellites were launched into space in 1978. The system was declared fully operational in every 12 hours, to provide worldwide position, time and velocity information. GPS makes it possible to precisely identify locations on the earth by measuring distance from the satellites. GPS allows you to record or create locations from places on the earth and helps to travel to different positions. It was first designed for military. Later on in 1980, it was used for civilian applications.

There are three segments of GPS.

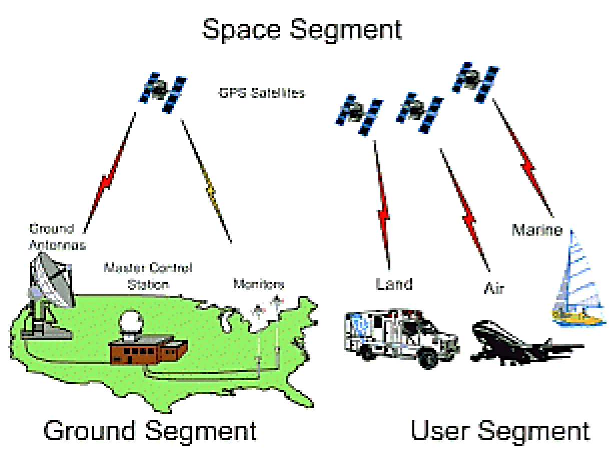


Figure 2.1: Segments of GPS

2.1.1 SPACE SEGMENT

The space segment consists of 24 satellites circling the earth at 12,000 miles in altitude. This high altitude allows the signals to cover a greater area. The satellites are arranged in their orbits, so a GPS receiver on earth can always receive a signal from at least four satellites at any given time. Each satellite transmits low radio signals with a unique code on different frequencies, allowing the GPS receiver to identify the signals. The main purpose of these coded signals is to allow for calculating travel time from the satellite to the GPS receiver. The travel time multiplied by the speed of light equals the distance from the satellite to the GPS receiver. Since these are low power signals and won't travel through solid objects, it is important to have a clear view of the sky.

2.1.2 CONTROL SEGMENT

The control segment tracks the satellites and then provides them with corrected orbital and time information. The control segment consists of four unmanned control stations and one master control station. The four unmanned stations receive data from the satellites and then send that information to the master control station where it is corrected and sent back to the GPS satellites.

2.1.3 USER SEGMENT

The user segment consists of the users and their GPS receivers. The number of simultaneous users is limitless.

2.2 GPS WORKING

When a GPS receiver is turned on, it first downloads orbit information of all the satellites. This process, the first time, can take as long as 12.5 minutes, but once this information is downloaded; it is stored in the receiver's memory for future use. Even though the GPS receiver knows the precise location of the satellites in space, it still needs to know the distance from each satellite it is receiving a signal from. That distance is calculated, by the receiver, by multiplying the velocity of the transmitted signal by the time it takes the signal to reach the receiver. The receiver already knows the velocity, which is the speed of a radio wave or 186,000 miles per second (the speed of light). To determine the time part of the formula, the receiver matches the satellites transmitted code to its own code, and by comparing them determines how much it needs to delay its code to match the satellites code. This delayed time is multiplied by the speed of light to get the distance. The GPS receiver's clock is less accurate than the atomic clock in the satellite. Therefore, each distance measurement must be corrected to account for the GPS receiver's internal clock error.

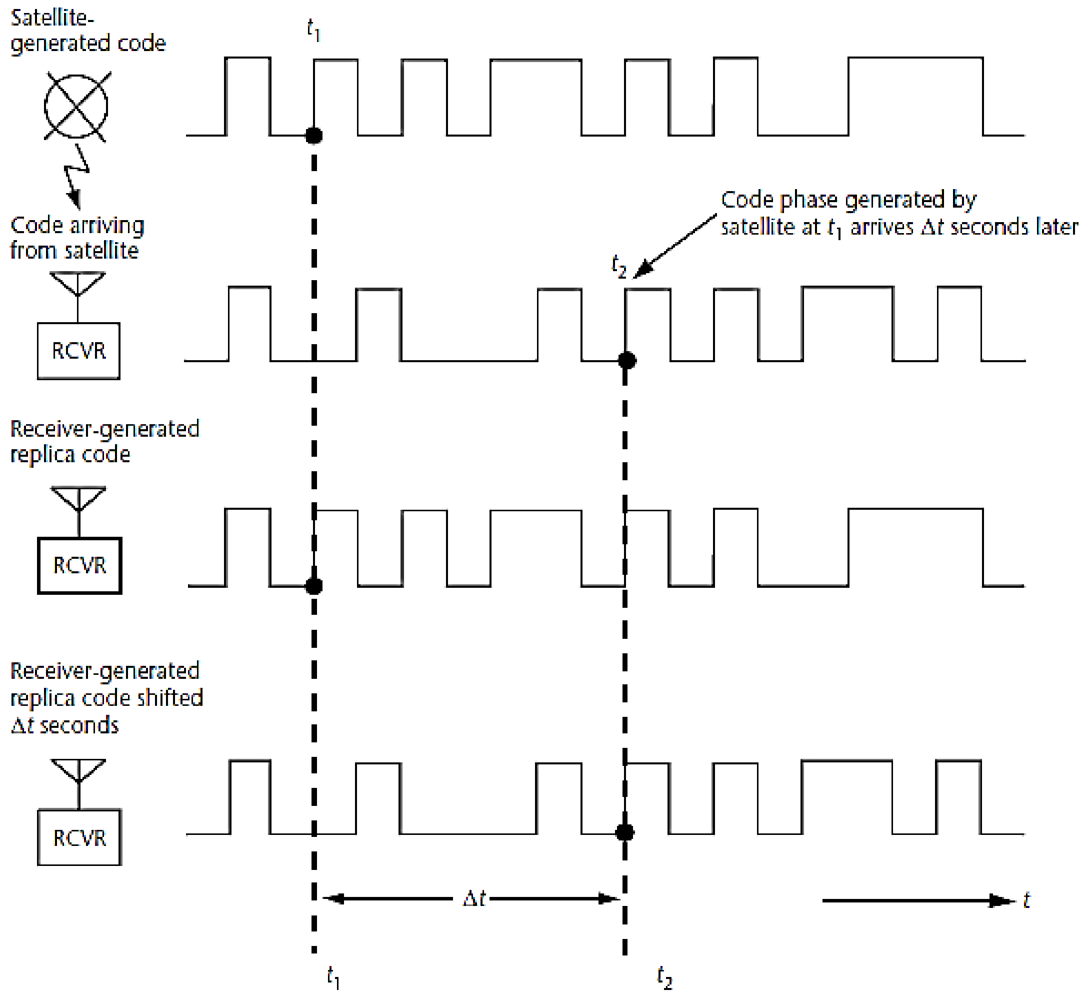


Figure 2.2: Use of replica code to determine satellite code transmission time.

2.2.1 PRINCIPLE OF TRILATERATION

Using three distances, trilateration can pinpoint a precise location. Each satellite is at the center of a sphere and where they all intersect is the position of the GPS receiver.

4 satellites transmit suitably coded signals which contain the ephemeris and status information of satellite. Based on this information, the receivers calculate the range to satellites, satellite coordinates and compute its location.

The figures shown below depict the principle of trilateration:

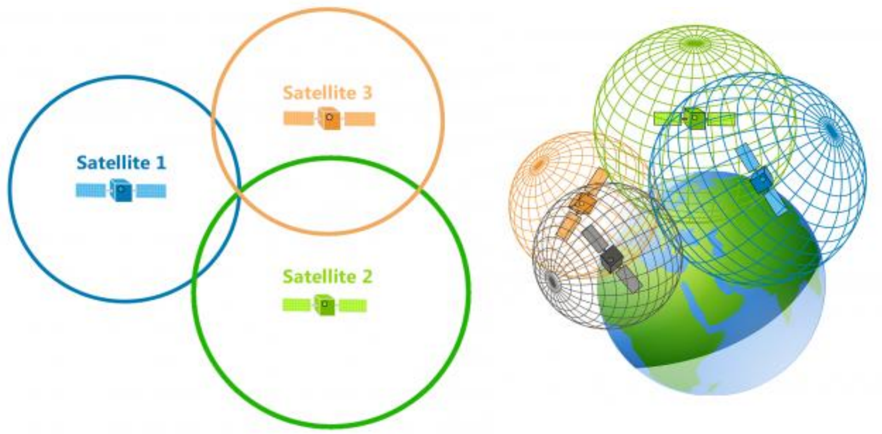


Figure 2.3: Principle of Trilateration

2.3 SOURCES OF GPS ERROR

The satellite signal is subject to various sources of error before reaching the receiver. These error sources degrade the signal decreasing the accuracy of the pseudo range and subsequently the obtained position estimate. The errors resulting from these sources which more or less a constant bias can be systematic whose effects persist over a longer period of time or there can be random sources of error which contribute to signal noise and rapid changes.

GPS receivers have potential position errors due to some of the following sources:

1. User errors
2. Satellite clock errors
3. Ionospheric errors
4. Tropospheric errors
5. Multipath errors
6. Satellite geometry

2.3.1 USER ERRORS

They account for most of the GPS errors. Incorrect datum and typographic errors when inputting coordinates into a GPS receiver can result in errors up to many kilometers. Unknowingly relying on less than four satellites for determining position coordinates can also result in unreliable position fixes that can easily be off by a distance in excess of a mile. Even the human body can cause signal interference. Holding a GPS receiver close to the body can block some satellite signals and hinder accurate positioning. If a GPS receiver must be hand held without benefit of an external antenna, facing to the south can help to alleviate signal blockage caused by the body because the majority of GPS satellites are oriented more in the earth's southern hemisphere. A GPS receiver has no way to identify and correct the user errors.

2.3.2 SATELLITE CLOCK ERRORS

Although all the satellites carry atomic clocks that are very accurate, even these can deviate from GPS system time (GPST), which results in pseudo range error.The Master Control Station determines the clock error of each satellite and transmits clock correction parameters to the satellites for rebroadcast of these in the navigation message. The receiver using these parameters implements correction.

2.3.3 ORBIT ERRORS

Satellite orbit (referred to as “satellite ephemeris”) pertains to the altitude, position and speed of the satellite. Satellite orbits vary due to gravitational pull and solar pressure fluctuations. Orbit errors are also monitored and corrected by the Master Control Station.

2.3.4 IONOSPHERIC ERRORS

The ionosphere is the layer of the atmosphere from 50 to 500 km altitude that consists primarily of ionized air. Ionospheric interference causes the GPS satellite radio signals to be refracted as they pass through the earth’s atmosphere – causing the signals to slow down or speed up. This results in inaccurate position measurements by GPS receivers on the ground. Even though the satellite signals contain correction information for ionospheric interference, it can only remove about half of the possible 70 nanoseconds of delay, leaving potentially up to a ten meter horizontal error on the ground. GPS receivers also attempt to “average” the amount of signal speed reduction caused by the atmosphere when they calculate a position fix. But this works only to a point. Fortunately, error caused by atmospheric conditions is usually less than 10 meters. This source of error has been further reduced with the aid of the Wide Area Augmentation System (WAAS), a space and ground based augmentation to the GPS.

2.3.5 TROPOSPHERIC ERRORS

The troposphere is the lower layer of the earth's atmosphere (below 13 km) that experiences the changes in temperature, pressure, and humidity associated with weather changes. GPS errors are largely due to water vapor in this layer of the atmosphere. Tropospheric interference is fairly insignificant to GPS. Receiver noise is simply the electromagnetic field that the receiver's internal electronics generate when it's turned on. Electromagnetic fields tend to distort radio waves. This affects the travel time of the GPS signals before they can be processed by the receiver. Remote antennas can help to alleviate this noise. This error cannot be corrected by the GPS receiver.

2.3.6 MULTIPATH INTERFERENCE

It is caused by reflected radio signals from surfaces near the GPS receiver that can either interfere with or be mistaken for the true signal that follows an uninterrupted path from a satellite.

An example of multipath is the ghosting image that appears on a TV equipped with rabbit ear antennas. Multipath is difficult to detect and sometimes impossible for the user to avoid, or for the receiver to correct. Common sources of multipath include car bodies, buildings, power lines and water.

When using GPS in a vehicle, placing an external antenna on the roof of the vehicle will eliminate most signal interference caused by the vehicle. Using a GPS receiver placed on the dashboard will always have some multipath interference.

2.3.7 SATELLITE GEOMETRY

This refers to the relative position of the satellites at any given time. Ideal satellite geometry exists when the satellites are located at wide angles relative to each other. Poor geometry exists when the satellites are located in a line or in a tight grouping.

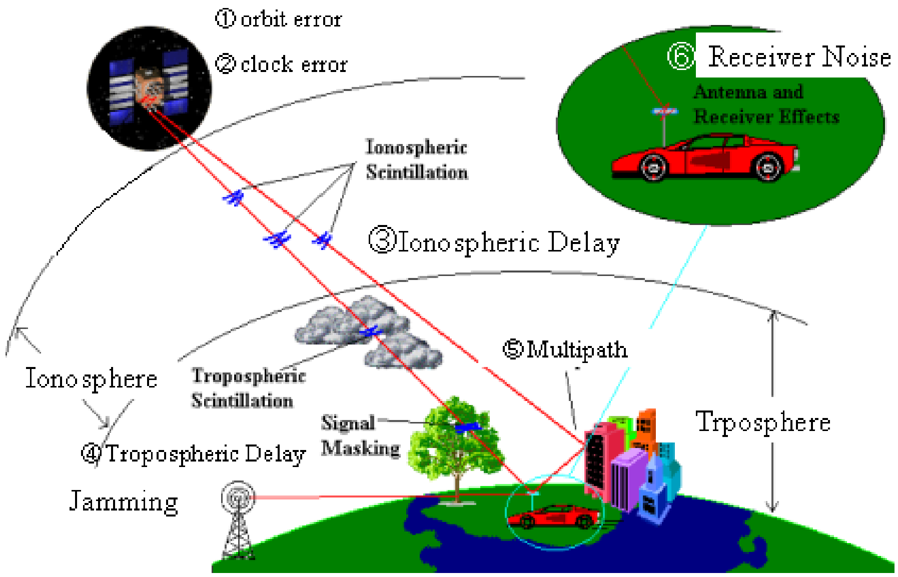


Figure 2.4: Errors on GPS signals

CHAPTER 3

INDIAN REGIONAL NAVIGATION SATELLITE SYSTEMS

3.1 BACKGROUND

In 1998, India started Operation Shakti (Pokhran-II) nuclear test, which was completely hidden from US spy satellites. This was a challenging task as the US spy satellites are capable of capturing even the minute details. India felt the importance of secrecy and to avoid being detected by other countries. The extensive and intelligent work of Indian scientists helped to prevent the detection of test sites from orbiting satellites of several countries. From the analysis of information about the orbiting satellite movement provided by scientists, Indian Army made sure that the satellite images do not have any trace of these movements. India successfully conducted the test without being detected, displaying India's capabilities in analyzing the moving satellites and the science of intelligence related to camouflage.

During the Kargil war in 1999, one of the things Indian military sought out was the American GPS data for that region. The Navigation System maintained by the US government would have provided vital information, but US denied. Understanding the inevitable need of navigation systems, later in May 2006, the IRNSS project was approved. Today, the Indian Space Research Organization (ISRO) took the nation closer to its goal by accomplishing the task of developing the country's own navigation system with the successful launch of IRNSS satellite constellation. With the launch of the satellites, India has joined the league of countries that has indigenous navigation system. This will reduce the country's dependency on US Global Positioning System.

There are four 'Global Navigation Satellite Systems (GNSS)' in the world. GPS (Global Positioning System) of United States, GLONASS (Global Satellite Navigation System) of Russia, Galileo of European Union and BeiDou of China. There are two 'Regional Satellite Navigation Systems', the QZSS (Quasi-Zenith Satellite System) of Japan and Compass of China.

Different navigation systems across the world are compared as shown below in table 3.1 and are mapped as shown in figure 3.1:

Table 3.1 Comparison of Navigation Systems

Navigation Systems Around the World				
Navigation systems	**Country**	**Operator**	**Type**	**Coverage**
Global Positioning system (GPS)	United States	Air Force Space Command (AFSPC)	Military ,civilian	Global
GLONASS	-Russia	Russian Aerospace Defense Forces, VKO	Military	Global
BeiDou Navigation Satellite system(BDS)	China	China National Space Administration(CNSA)	Military, Commercial	Global Operational (regionally)
Indian Regional Navigation Satellite System(IRNSS)	India	Indian Space Research Organization (ISRO)	Military, Civilian	Regional
Galileo(in development)	European Union	GSA, ESA	Civilian, Commercial	Global
Quasi-Zenith Satellite System(QZSS)-(in development)	Japan	Japan Aerospace exploration Agency(JAXA)	Civilian	Regional

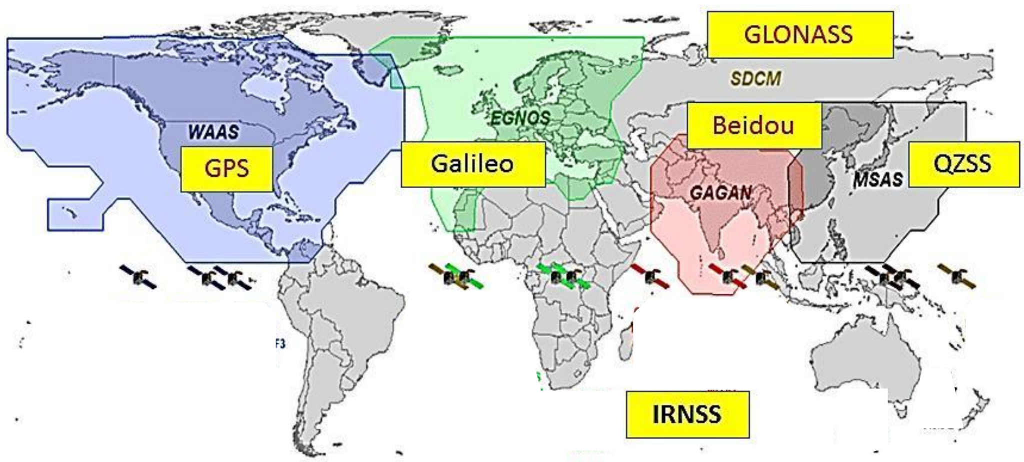


Figure 3.1: Global Navigation Satellite Systems

3.2 STRATEGIC IMPORTANCE

IRNSS, the indigenous navigation system will aid the terrestrial, aerial and marine navigation, vehicle tracking and fleet management, disaster management, mapping and geodetic data capture. The restricted service will be used by the military. IRNSS will ensure C4ISR (Command, Control, Computers, Communications, Intelligence, Surveillance and Reconnaissance). Hence, it will help in integrating data fusion and integration management system, radar, infrared, space based surveillance ensuring a high level accuracy in modern warfare.

Most importantly IRNSS satellites are placed in High Earth Orbit (HEO) at a height of 35,786 kilometers. It has importance when considering anti-satellite missiles. It makes IRNSS out of range of solid-fueled intercontinental missiles.

3.3 DEVELOPMENT

IRNSS is an autonomous regional satellite navigation system being developed by ISRO (Indian Space Research Organization), approved by Government of India in May 2006. The objective of the project is to implement an independent and indigenous regional space borne navigation system for national applications. The IRNSS is being developed parallel to the GAGAN (GPS Aided GEO Augmented Satellite Navigation) program.

As part of the project, the Indian Space Research Organization (ISRO) opened a new satellite navigation center within the campus of ISRO-Deep Space Network (DSN), Karnataka on 28 May 2013. A network of 21 ranging stations located across the country will provide data for monitoring of the satellites and navigation signal with complete control, including the space segment, ground segment and user receivers all being built in India. Missile targeting could be an important military application for the constellation. The total cost of the project is expected to be ₹1,420 crore (US$217 million).

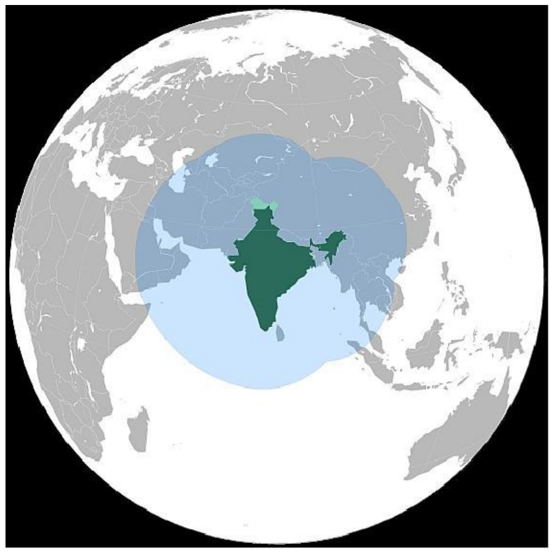


Figure 3.2: IRNSS Coverage

3.4 ARCHITECTURE

The IRNSS design requirements call for a position accuracy of less than 20 m throughout India and within the region of coverage extending about 1500 km of radius. The system is expected to provide accurate real-time position, velocity and time observable for users on a variety of platforms with 24 hour x 7 day service availability under all weather conditions. The proposed IRNSS system will consist of a constellation of seven satellites and a supporting ground segment. Three of the satellites in the constellation will be placed in a geostationary orbit and the remaining four in a geosynchronous inclined orbit of 29º relative to the equatorial plane. Such an arrangement would mean all seven satellites would have continuous radio visibility with Indian control stations.

The IRNSS constellation architecture consists of the following elements:

• Space segment

• Ground segment

• User segment

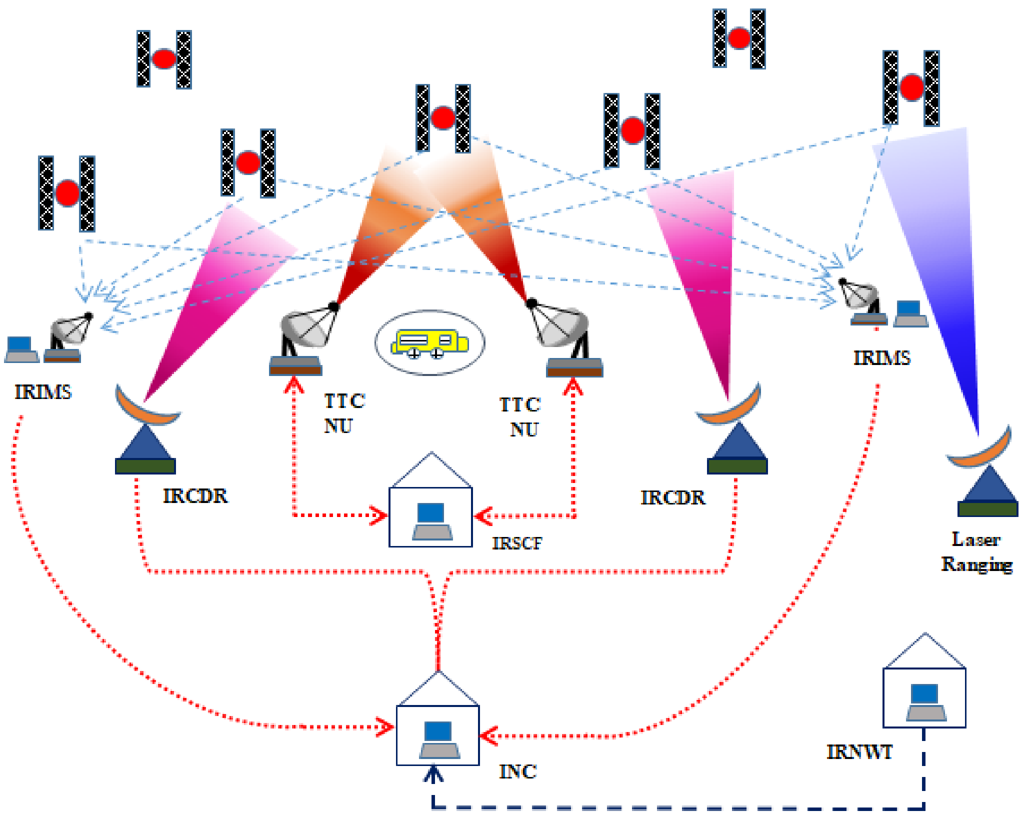


Figure 3.3: IRNSS architecture

3.4.1 SPACE SEGMENT

The constellation consists of 7 satellites. Among the seven satellites, three of them are located in geostationary orbit (GEO) at 32.5° East, 83° East, and 131.5° East longitude, approximately 36,000 km (22,000 mi) above earth surface. Remaining four satellites are in inclined geosynchronous orbit (GSO). Two of them cross equator at 55° East and two at 111.75° East. The four GSO satellites will appear to be moving in the form of an "8". Two spare satellites were also planned**.** The satellites are specially configured for the navigation with the same configuration for GEO and GSO which is desirable for the production of the satellites.

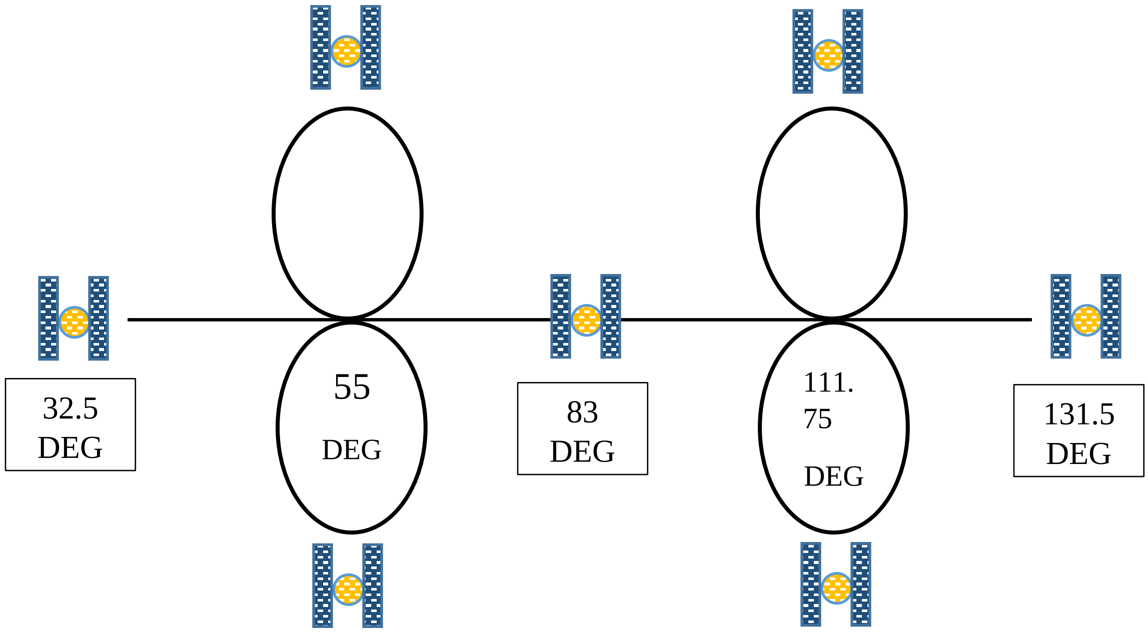


Figure 3.4: IRNSS Constellation

3.4.2 GROUND SEGMENT

Ground Segment is responsible for the maintenance and operation of the IRNSS constellation.

The Ground segment comprises of:

- IRNSS Spacecraft Control Facility (IRSCF)
- ISRO Navigation Centre (INC)
- IRNSS Range and Integrity Monitoring Stations (IRIMS)
- IRNSS Network Timing Centre (IRNWT)
- IRNSS CDMA Ranging Stations (IRCDR)
- Laser Ranging Stations
- IRNSS Data Communication Network(IRDCN)

The INC established, performs remote operations and data collection with all the ground stations. 14 IRIMS are currently operational and are supporting IRNSS operations. CDMA ranging is being carried out by the four IRCDR stations on regular basis for all the IRNSS satellites. The IRNWT has been established and is providing IRNSS system time with an accuracy of 20ns (2 sigma) w.r.t UTC. Laser ranging is being carried out with the support of ILRS stations around the world. Navigation Software is operational at INC since 1st Aug, 2013.

All the navigation parameters viz. satellite ephemeris, clock corrections, integrity parameters and secondary parameters viz. ionospheric-delay corrections, time offsets w.r.t UTC and other GNSS, almanac, text message and earth orientation parameters are generated and up-linked to the spacecraft automatically. The IRDCN has established terrestrial and VSAT links between the ground stations.

3.4.3 USER SEGMENT

The user segment mainly consists of:

- Single frequency IRNSS receiver capable of receiving of receiving SPS signal at L5 or S band frequency.
- A dual frequency IRNSS receiver capable of receiving both L5 and S band frequencies.
- A receiver compatible to IRNSS and other GNSS signals.

They will be able to receive and process navigation data from other GNSS constellations and the 7 IRNSS satellites will be continuously tracked by the user receivers. The user receiver will have a minimum gain G/T of -27 dB/K.

Figure specifies the radio frequency interface between space and user segments. Each IRNSS satellite provides SPS signals in L5 and S bands.

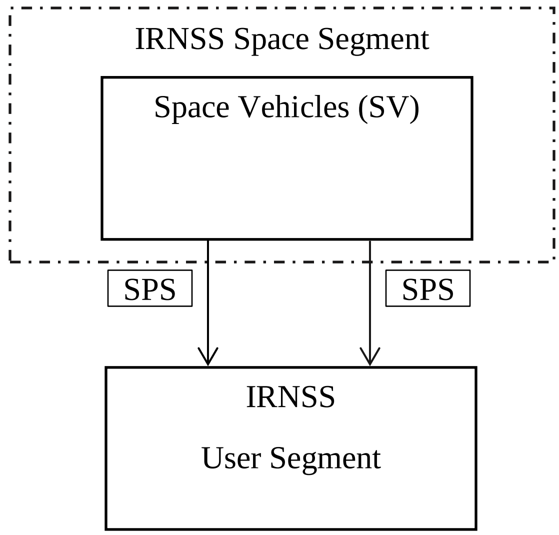


Figure 3.5: Space segment interface with User segment

3.5 IRNSS SIGNALS

3.5.1 FREQUENCY BANDS

The IRNSS SPS service is transmitted on L5 (1164.45 – 1188.45 MHz) and S (2483.5-2500 MHz) bands. The frequency in L5 band has been selected in the allocated spectrum of Radio Navigation Satellite Services and S band.

3.5.2 IRNSS SERVICES

There will be two kinds of services:

- Special Positioning Service (SPS)- An open service without encryption
- Precision Service (PS)- An authorized service with encryption

Both services will be carried on L5 (1176.45 MHz) and S band (2492.08 MHz). The navigation signals would be transmitted in the S-band frequency (2–4 GHz) and broadcast through a phased array antenna to keep required coverage and signal strength.

3.6 IRNSS SATELLITES

The IRNSS satellites with a dry mass of approximately 600 kg and a launch mass of 1,425 kg are configured around the spacecraft bus I-1K. The solar panels generate a power of 1600W. The spacecraft are 3-axis stabilized. Attitude control of the satellite is provided with yaw steering, a capability to optimize the use of the solar panels and to support the thermal control of the satellite.

The satellite platform is three-axis stabilized using a zero momentum system consisting of reaction wheels, magnetic torquers as well as attitude control thrusters. The satellite provides precise pointing capability. The main propulsion system for large orbit adjustments and apogee maneuvers consists of a Liquid Apogee Motor, LAM. The propellants are stored in spherical tanks that are pressurized with helium.

The heart of the satellite payload is a highly accurate rubidium atom clock that is used to generate navigation signals with two additional clocks available as backup. The payload of the satellite operates in L5-band at a center frequency of 1176.45 MHz and a bandwidth of 24MHz and in S-Band at 2492.028 MHz with a bandwidth of 16.5MHz.

Each IRNSS satellite has a life expectancy of 10 years. Once fully deployed, the IRNSS constellation will provide navigation service to India and surrounding areas as far as 1,500Km.

The constellation consists of 7 active satellites. Three of the seven satellites in constellation are located in geostationary orbit (GEO) and four in inclined geosynchronous orbit (GSO).
All satellites launched or proposed for the system are shown in table 3.2:

Table 3.2: Details of the IRNSS satellite constellations

IRNSS SATELLITES					
PROJECT	**LAUNCH DATE**	**LAUNCH VEHICLE**	**ORBIT**	**PAYLOAD**	**STATUS**
IRNSS-1A	1ST Jul 2013	PSLV-C22	Geosynchronous / 55°E, 29° inclined orbit	Navigation payload in S band and L5 band, CDMA ranging payload in C band, Rubidium Atomic Frequency Standard	Failed in orbit
IRNSS-1B	4th Apr 2014	PSLV-C24	Geosynchronous / 55°E, 29° inclined orbit	Navigation payload in S band and L5 band, CDMA ranging payload in C band, Rubidium Atomic Frequency Standard	Operational
IRNSS-1C	15th Oct 2014	PSLV-C26	Geostationary / 83°E, 5° inclined orbit	Navigation payload in S band and L5 band, CDMA ranging payload in C band, Rubidium Atomic Frequency Standard	Operational
IRNSS-1D	28th Mar 2015	PSLV-C27	Geosynchronous / 111.75°E, 31° inclined orbit	Navigation payload in S band and L5 band, CDMA ranging payload in C band, Rubidium Atomic Frequency Standard	Operational

IRNSS-1E	20th Jan 2016	PSLV-C31	Geosynchronous / 111.75°E, 29° inclined orbit	Navigation payload in S band and L5 band, CDMA ranging payload in C band, Rubidium Atomic Frequency Standard	Operational
IRNSS-1F	10th Mar 2016	PSLV-C32	Geostationary / 32.5°E, 5° inclined orbit	Navigation payload in S band and L5 band, CDMA ranging payload in C band, Rubidium Atomic Frequency Standard	Operational
IRNSS-1G	28th Apr 2016	PSLV-C33	Geostationary / 129.5°E, 5.1° inclined orbit	Navigation payload in S band and L5 band, CDMA ranging payload in C band, Rubidium Atomic Frequency Standard	Operational
IRNSS-1H	31st Aug 2017	PSLV-C39	Geosynchronous / 55°E, 29° inclined orbit	Navigation payload in S band and L5 band, CDMA ranging payload in C band, Rubidium Atomic Frequency Standard	Launch failed
IRNSS-1I	Mar/Apr 2018				Under Development

3.7 IRNSS RECEIVER

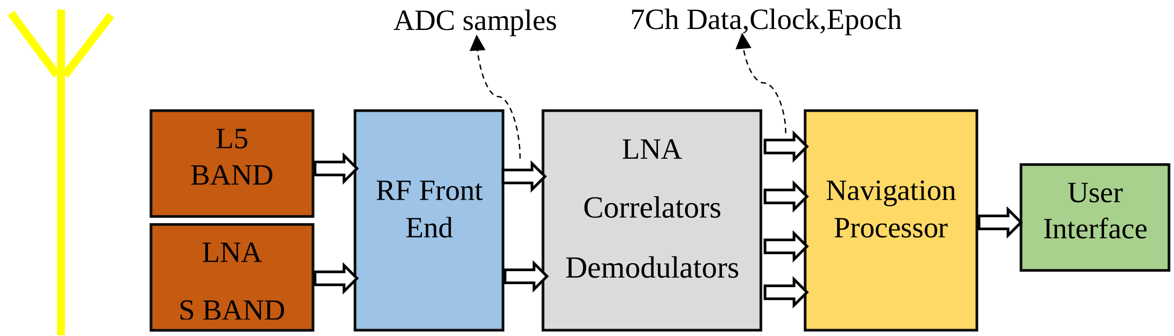


Figure 3.6: IRNSS Receiver

The above IRNSS receiver block diagram (shown in figure 3.6) explains about the various processes involved in receiving IRNSS signals. There are two types of user receivers:

- Single Frequency
  - L-5 Band: BPSK (Civil Use) Or BOC (Restricted)
  - S Band : BPSK (Civil Use) Or BOC (Restricted)
- Dual Frequency
  - BPSK(Civil Use) Or BOC (Restricted)

The block diagram consists of:

- Antenna
  - Band patch antenna for reception
  - Dual band antenna for reception
- Low Noise Amplifier(LNA)
  - Performance of low power with high gain amplifier
- RF Front End Processor
  - Re-configurable frequency conversion device
- Correlator & Demodulator
  - Forward Error Correction (FEC), interleaving, BOC(5,2)
  - BPSK modulation and demodulation and interpolate signals

- Navigation Processor & User Interface
    - Frame extraction, decryption (for RS service only), pseudo range calculation
    - Ionospheric/Tropospheric corrections
    - User position calculation

3.8 NAVIGATION DATA

The navigation data (NAV data) includes IRNSS satellite ephemeris, IRNSS time, satellite clock correction parameters, status messages and other secondary information etc. Navigation data modulated on top of the ranging codes can be identified as primary and secondary navigation parameters:

Primary Navigation Parameters:

- Satellite ephemeris
- Satellite clock correction parameters
- Satellite & signal health status
- User range accuracy
- Total group delay

Secondary Navigation Parameters:

- Satellite almanac
- Ionospheric grid delays and confidence
- IRNSS time offsets with respect to UTC & GNSS
- Ionospheric delay correction coefficients
- Text messages
- Differential corrections
- Earth orientation parameters

3.8.1 NAVIGATION DATA FORMATS

Global Navigation Satellite System (GNSS) formats have been developed within the government, industry and academia. Their standardization has facilitated the efficient development of the GNSS industry. Current GNSS formats support meta-data such as GNSS station, receiver, antenna parameters, equipment calibration information, GNSS observation, broadcast navigation information and also GNSS products such as precise orbits, clock corrections, atmospheric measurements and station coordinates. The GNSS community relies on a variety of standards that have been developed by different entities to facilitate an interoperable and efficient exchange of data and products between providers and users. The most widely used GNSS standards are:

- RTCM SC-104 standard for Differential GNSS Services of the Radio Technical Commission for Maritime Services (RTCM).
- The GNSS-related parts of the NMEA 0183 interface standard of the National Marine Electronics Association (NMEA).
- The Receiver INdependent EXchange (RINEX) format developed by the International GNSS Service (IGS and RTCM SC-104).

3.9 RINEX FORMAT

Receiver INdependent EXchange Format (RINEX) is a data interchange format for raw satellite navigation system data. This allows the user to post-process the received data to produce a more accurate result — usually with other data unknown to the original receiver, such as better models of the atmospheric conditions at time of measurement. The final output of a navigation receiver is usually its position, speed or other related physical quantities.

3.9.1 HISTORY OF RINEX

The first proposal for the ***Receiver Independent Exchange Format RINEX*** was developed by the Astronomical Institute of the University of Bern for the easy exchange of the Global Positioning System (GPS) data to be collected during the first large European GPS campaign EUREF 89, which involved more than 60 GPS receivers of 4 different manufacturers. The governing aspect during the development was that, most geodetic processing software for GPS data use a well-defined set of observables:

- Carrier-phase measurement at one or both carriers (a measurement on the beat frequency between the received carrier of the satellite signal and a receiver-generated reference frequency).

- Pseudo range (code) measurement, equivalent to the difference of the time of reception (expressed in the time frame of the receiver) and the time of transmission (expressed in the time frame of the satellite) of a distinct satellite signal.
- Observation time (the reading of the receiver clock at the instant of validity of the carrier-phase and/or the code measurements).

3.9.2 FORMAT DESCRIPTION

The RINEX version 3.XX format consists of three ASCII file types:

1. Observation data file
2. Navigation message file
3. Meteorological data file

Each file type consists of a header section and a data section. The header section contains global information for the entire file and is placed at the beginning of the file. The header section contains header labels in columns 61-80 for each line contained in the header section. These labels are mandatory and must appear exactly. Each Observation file and each Meteorological Data file basically contain the data from one site and one session.

RINEX version 3 navigation message files may contain navigation messages of more than one satellite system (GPS, GLONASS, Galileo, Quasi Zenith Satellite System (QZSS), BeiDou System (BDS), Indian Regional Navigation Satellite System (IRNSS) and SBAS.

Observation data file

- RINEX 3 observation data files are made up of a variable-length header followed by a series observation record providing the measurements at each epoch.

Navigation message file

- As a complement to the observation data format, the RINEX standard also defines navigation data format for the exchange of broadcast ephemeris information, comprising of the orbit and clock parameters as well as auxiliary data transmitted by each GNSS satellite for use in real-time navigation.

Meteorological data file

- Meteorological data file basically contain the data from one site and one session.

3.9.3 GNSS OBSERVABLES

GNSS observables include three fundamental quantities: Time, Phase, and Range.

1. Time

The time of the measurement is the receiver time of the received signals. It is identical for the phase and range measurements and is identical for all satellites observed at that epoch. For single-system data files, it is by default expressed in the time system of the respective satellite system. For mixed files, the actual time system used must be indicated in the TIME OF FIRST OBS header record.

2. Pseudo-Range

The pseudo-range (PR) is the distance from the receiver antenna to the satellite antenna including receiver and satellite clock offsets (and other biases, such as atmospheric delays):

PR = distance + c * (receiver clock offset –satellite clock offset + other biases)

So, the pseudo-range reflects the actual behavior of the receiver and satellite clocks. The pseudo-range is stored in units of meters.

3. Phase

The phase is the carrier-phase measured in whole cycles. The half-cycles measured by squaring-type receivers must be converted to whole cycles and flagged by the respective observation code.

The phase changes in the same sense as the range (negative Doppler). The phase observations between epochs must be connected by including the integer number of cycles. The observables are not corrected for external effects such as: atmospheric refraction, satellite clock offsets, etc. If necessary, phase observations are corrected for phase shifts needed to guarantee consistency between phases of the same frequency and satellite system based on different signal channels.

4. Doppler

The sign of the Doppler shift as additional observable is positive for approaching satellites and negative for receding satellites.

5. Satellite numbers

The former two-digit satellite numbers are preceded by a one-character system identifier.The same satellite system identifiers are also used in all header records when appropriate.

```
     3.03           N:IRNSS NAV DATA      I: IRNSS          RINEX VERSION / TYPE
IRNSS NAV GEN       ACCORD              20-SEP-16 14:25     PGM / RUN BY / DATE
Version Number of DataExtraction_IRNSSUR : 1.16             COMMENT
IRNA   1.3970D-08 -2.7567D-07  7.5102D-06 -7.5102D-06       IONOSPHERIC CORR
IRNB  -0.2601D+06  0.2081D+07 -0.8323D+07  0.8323D+07       IONOSPHERIC CORR
IRUT - 2.4068867788D-08 7.993605777D-15  86688  888         TIME SYSTEM CORR
    17                                                      LEAP SECONDS
                                                            END OF HEADER
I05 2016 08 29 18 30 24 7.925797253847D-04 9.208633855451D-12 0.000000000000D+00
     0.900000000000D+01-0.648812500000D+03 2.881548599494D-09 0.104816112901D+01
    -2.102926373482D-05 1.821160432883D-03 1.788139343262D-05 0.649343601990D+04
     0.151200000000D+06-4.470348358154D-08 0.250092196763D+01-1.415610313416D-07
     4.950972088432D-01-0.459312500000D+03-0.313808188427D+01-2.480103306258D-09
     8.143196339671D-11                    0.888000000000D+03
     0.200000000000D+01 0.000000000000D+00-4.656612873077D-10
     0.144000000000D+06
I07 2016 08 29 18 30 24 4.021725617349D-04 6.764366844436D-11 0.000000000000D+00
     0.900000000000D+01-0.740125000000D+03 8.017476817234D-09-0.195091748023D+01
    -2.427026629448D-05 1.986416755244D-04-2.149492502213D-06 0.649329118729D+04
     0.151200000000D+06-5.215406417847D-08-0.111722637952D+01-2.086162567139D-07
     8.410532549583D-02 0.753125000000D+02-0.249552429146D+01-6.903144686542D-09
    -4.968064082668D-10                    0.888000000000D+03
     0.200000000000D+01 0.000000000000D+00-1.396983861923D-09
     0.144000000000D+06
I02 2016 08 29 18 30 24 8.126189932227D-04-1.853095454862D-11 0.000000000000D+00
     0.900000000000D+01-0.189187500000D+03 2.771544017362D-09-0.312870674011D+01
    -6.034970283508D-06 2.221348229796D-03 2.058222889900D-05 0.649333260536D+04
     0.151200000000D+06 1.117587089539D-08-6.469914897697D-01 1.154839992523D-07
     5.167321931357D-01-0.540937500000D+03-0.308729596631D+01-2.715827410827D-09
    -6.682421206809D-10                    0.888000000000D+03
     0.200000000000D+01 0.000000000000D+00-1.862645149231D-09
     0.144000000000D+06
```

Figure 3.7: Sample RINEX Navigation Data file

Figure 3.7 shows a sample of RINEX navigation file that is used for simulation of this project. The sample contains both header and data sections; data section includes the ephemeris information of three IRNSS satellites with PRN numbers 5, 7 and 2.

```
     3.03           OBSERVATION DATA     M:MIXED             RINEX VERSION / TYPE
OBS GEN             ACCORD              20-SEP-16 14:25      PGM / RUN BY / DATE
Version Number of DataExtraction_IRNSSUR : 1.16              COMMENT
Unknown                                                      MARKER NAME
0                                                            MARKER NUMBER
HUMAN                                                        MARKER TYPE
Unknown             Unknown                                  OBSERVER / AGENCY
100883              18700               17040131             REC # / TYPE / VERS
18730               L5L1S1                                   ANT # / TYPE
        0.0000        0.0000        0.0000                   APPROX POSITION XYZ
        0.0000        0.0000        0.0000                   ANTENNA: DELTA H/E/N
G    4 C1C L1C D1C S1C                                       SYS / # / OBS TYPES
I    8 C5C L5C D5C S5C C9C L9C D9C S9C                       SYS / # / OBS TYPES
DBHZ                                                         SIGNAL STRENGTH UNIT
SNR is mapped to RINEX snr flag value [1-9]                  COMMENT
>= 25dBHz -> 1; 26-27dBHz -> 2; 28-31dBHz -> 3               COMMENT
32-35dBHz -> 4; 36-38dBHz -> 5; 39-41dBHz -> 6               COMMENT
42-44dBHz -> 7; 45-48dBHz -> 8; >= 49dBHz -> 9               COMMENT
     1.000                                                   INTERVAL
  2016     8    29    18    30   18.0000000     GPS          TIME OF FIRST OBS
  2016     8    30    18    29   43.0000000     GPS          TIME OF LAST OBS
G L1C                                                        SYS / PHASE SHIFT
I L5A                                                        SYS / PHASE SHIFT
I L9A                                                        SYS / PHASE SHIFT
                                                             END OF HEADER
> 2016 08 29 18 30 18.0000000  0 19
G04  21070510.481 8 110726292.128 8       150.263 8        47.470
G31  21752721.453 8 114311399.379 8     -2559.332 8        47.850
G11  24360529.532 5 128015478.260 5     -1499.139 5        38.421
G27  25245549.648 6 132666264.684 6      1014.743 6        39.871
G22  21348419.691 8 112186720.341 8      2285.889 8        47.867
G26  20997652.815 9 110343447.886 9      1068.327 9        49.796
G16  21752441.963 8 114309897.394 8      2974.587 8        47.301
G01  22729537.613 7 119444527.584 7      -866.195 7        43.588
G14  22629071.903 7 118916669.096 7      -459.959 7        44.048
G32  23723429.200 7 124667580.256 7     -1130.816 7        42.410
G10  23876803.654 7 125473560.956 7     -3207.888 7        43.111
G23  25093436.974 7 131866911.228 7      1694.118 7        42.720
```

Figure 3.8: Sample RINEX Observation Data file

Figure 3.8 shows a sample of RINEX observation file that is used for simulation of this project. The sample contains both header and data sections, data section includes the GPS measurement information from the GPS satellites with PRN numbers 4,31,11,27,22,26,16,1,14,32,10,and 23.

CHAPTER 4

ALGORITHMS FOR USER POSITION DETERMINATION

4.1 INTRODUCTION

The computation of GPS user position involves solving a system of non-linear Euclidean system of distance equations. The equations are according to the principle of trilateration, three unknowns in 3 equations, but we require at least 4 equations for determination of clock bias apart from 3 ECEF coordinates.

$$\boldsymbol{\rho_i = \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2}} \; \textbf{.... (4.1)}$$

The equation (**4.1**) represents the Euclidean distance equation from **i**$^{\text{th}}$ ***(i = 1, 2, 3 and 4)*** satellite to the user **u** on the surface of earth.

The pseudo range is measured using the following equations:

$$\boldsymbol{\rho_i = \rho_{iT} + \Delta D_i - c(\Delta b_i - b_{ut}) + c(\Delta T_i + \Delta I_i + v_i + \Delta v_i)} \; \textbf{.... (4.2)}$$

where $\Delta \boldsymbol{D_i}$ the satellite position error effect on range is, $\Delta \boldsymbol{T_i}$ is the tropospheric delay error, $\Delta \boldsymbol{I_i}$ is the ionospheric delay error, $\boldsymbol{v_i}$ is the receiver measurement noise error, $\Delta \boldsymbol{v_i}$ is the relativistic time correction.

$$\boldsymbol{\rho_{iT} = c(t_u - t_{si})} \; \textbf{.... (4.3)}$$

Where ***c*** is the speed of light, $\boldsymbol{\rho_{iT}}$ is often referred to as the true value of pseudo range from user to satellite ***i***, $\boldsymbol{t_{si}}$ is referred to as the true time of transmission from satellite ***i***, $\boldsymbol{t_u}$ is the true time of reception.

The complete set of pseudo range equations are shown below:

$$\boldsymbol{\rho_1 = \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_u)^2} + b_u}$$

$$\boldsymbol{\rho_2 = \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2} + b_u}$$

$$\boldsymbol{\rho_3 = \sqrt{(x_3 - x_u)^2 + (y_3 - y_u)^2 + (z_3 - z_u)^2} + b_u}$$

$$\boldsymbol{\rho_4} = \sqrt{(\boldsymbol{x_1} - \boldsymbol{x_u})^2 + (\boldsymbol{y_1} - \boldsymbol{y_u})^2 + (\boldsymbol{z_1} - \boldsymbol{z_u})^2} + \boldsymbol{b_u}$$

4.2 LEAST MEAN SQUARES ALGORITHM

This algorithm involves iterative linearization of pseudo-range equations using Newton Raphson method, about a neighborhood point successively until the mean square error converges to a minimum value (it generally takes 5 to 6 iterations to converge to a solution).

On differentiating the pseudo range equations (4.1), we get the following linearized equation:

$$\delta\boldsymbol{\rho_i} = \frac{(\boldsymbol{x_i}-\boldsymbol{x_u})\delta\boldsymbol{x_u}+(\boldsymbol{y_i}-\boldsymbol{y_u})\delta\boldsymbol{y_u}+(\boldsymbol{z_i}-\boldsymbol{z_u})\delta\boldsymbol{z_u}}{\sqrt{(\boldsymbol{x_i}-\boldsymbol{x_u})^2+(\boldsymbol{y_i}-\boldsymbol{y_u})^2+(\boldsymbol{z_i}-\boldsymbol{z_u})^2}} + \delta\boldsymbol{b_u} \quad \textbf{.... (4.4)}$$

The above equations can be shown as a system of equations (Matrix Form):

$$\begin{bmatrix} \delta\rho_1 \\ \delta\rho_2 \\ \delta\rho_3 \\ \delta\rho_4 \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & 1 \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & 1 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & 1 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & 1 \end{bmatrix} \begin{bmatrix} \delta x_u \\ \delta y_u \\ \delta z_u \\ \delta b_u \end{bmatrix}$$

Where $\alpha_{i1} = \frac{x_i - x_u}{\rho_i - b_u}$ $\alpha_{i2} = \frac{y_i - y_u}{\rho_i - b_u}$ $\alpha_{i3} = \frac{z_i - z_u}{\rho_i - b_u}$

The above system of equation gives the solution for differential position coordinates, which are added to the initial position estimate (initial neighborhood).

Let us represent the above system as **Ax=b;**

For four satellites in view, the solution is given by **x=**$\boldsymbol{A^{-1}}$**b**

If more than four satellites are in the view of receiver the solution is given by least squares solution that is

$$\mathbf{x} = (\boldsymbol{A^T A})^{-1}\boldsymbol{A^T}\mathbf{b}$$

This minimizes the squared error value, $v = \sqrt{\delta x^2 + \delta y^2 + \delta z^2}$ which gives the approximate differential coordinate solution.

4.3 BANCROFT ALGORITHM

The Bancroft method allows obtaining a direct solution of the receiver position and the clock offset without requesting any "a priori" knowledge for the receiver location.

By expanding the pseudo range equations and converting it into matrix equations and then using linear algebra, we solve for receiver position and clock bias.

The pseudo-range equations are expanded by squaring the terms. The expanded equation for more than four satellites is expressed in terms of matrices and Lorentz inner product which results in a quadratic equation as shown below:

$$\langle \boldsymbol{B^{-1}1}, \boldsymbol{B^{-1}1} \rangle \boldsymbol{\Lambda^2} + \boldsymbol{2}[\langle \boldsymbol{B^{-1}1}, \boldsymbol{B^{-1}a} \rangle - \boldsymbol{1}]\boldsymbol{\Lambda} + \langle \boldsymbol{B^{-1}a}, \boldsymbol{B^{-1}a} \rangle = \boldsymbol{0}$$

The above equation is for four satellites. If the receiver is locked onto more than four satellites, the equation will be in terms of least squares as shown below:

$$\begin{aligned} &\langle (\boldsymbol{B^TB})^{-1}\boldsymbol{B^T1}, (\boldsymbol{B^TB})^{-1}\boldsymbol{B^T1} \rangle \boldsymbol{\Lambda^2} + \boldsymbol{2}[\langle (\boldsymbol{B^TB})^{-1}\boldsymbol{B^T1}, (\boldsymbol{B^TB})^{-1}\boldsymbol{B^Ta} \rangle - \boldsymbol{1}]\boldsymbol{\Lambda} \\ &+ \langle (\boldsymbol{B^TB})^{-1}\boldsymbol{B^Ta}, (\boldsymbol{B^TB})^{-1}\boldsymbol{B^Ta} \rangle = \boldsymbol{0} \end{aligned}$$

Where

$$B = \begin{bmatrix} x_1 & y_1 & z_1 & \rho_1 \\ x_2 & y_2 & z_2 & \rho_2 \\ x_3 & y_3 & z_3 & \rho_3 \\ x_4 & y_4 & z_4 & \rho_4 \end{bmatrix}, \ \boldsymbol{\rho_i} = \sqrt{(\boldsymbol{x_i} - \boldsymbol{x_u})^2 + (\boldsymbol{y_i} - \boldsymbol{y_u})^2 + (\boldsymbol{z_i} - \boldsymbol{z_u})^2} + \boldsymbol{b_u}$$

$$r = \begin{bmatrix} x_u \\ y_u \\ z_u \end{bmatrix}, \Lambda = \tfrac{1}{2}\langle \begin{bmatrix} r \\ b_u \end{bmatrix}, \begin{bmatrix} r \\ b_u \end{bmatrix} \rangle \ , 1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \ and \ a_i = \tfrac{1}{2}\langle \begin{bmatrix} r_i \\ \boldsymbol{\rho_i} \end{bmatrix}, \begin{bmatrix} r_i \\ \boldsymbol{\rho_i} \end{bmatrix} \rangle$$

On solving the above quadratic equation we get two solutions- One is on the earth’s surface and the other away from the earth’s surface.

4.4 RECURSIVE LEAST SQUARES ALGORITHM

The objective of this algorithm is to minimize the mean squared error between actual and estimated data, thus it provides the best estimate of data with respect to mean squared error. This algorithm takes the initial estimates as well as present measurements and updates the co-variance and projects it into the next iteration, hence updates the estimates thereby giving the accurate solution.

This algorithm solves for the differential position coordinates in the equation, **Ax=b,** by iteratively updating the coefficient matrix **A** with the incoming satellite observations and adds it to the initial position estimate and finally converges to the position estimate.

For the coefficient matrix formed at kth epoch the solution is given by

$$\boldsymbol{x_k}=(\boldsymbol{A^T}_{\boldsymbol{k}}\boldsymbol{A_k})^{-1}\boldsymbol{A^T}_{\boldsymbol{k}}\boldsymbol{b_k}$$

The recursive update formula is given by:

$$\boldsymbol{x_{k+1}} = \boldsymbol{x_k} + \boldsymbol{P_{k+1}}\boldsymbol{a_{k+1}}(\boldsymbol{R_{k+1}} - \boldsymbol{a^T}_{\boldsymbol{k+1}}\boldsymbol{x_k})$$

Where $\boldsymbol{P_{k+1}} = \boldsymbol{P_k} - \boldsymbol{P_k}\boldsymbol{a_{k+1}}[\boldsymbol{a^T}_{\boldsymbol{k+1}}\boldsymbol{P_k}\boldsymbol{a_{k+1}} + \mathbf{1}]^{-1}\boldsymbol{a^T}_{\boldsymbol{k+1}}\boldsymbol{P_k}$,

$\boldsymbol{P_k} = (\boldsymbol{A^T}_{\boldsymbol{k}}\boldsymbol{A_k})^{-1}$, $\boldsymbol{a_{k+1}} = \boldsymbol{updated\ coeffeient\ vector,}$

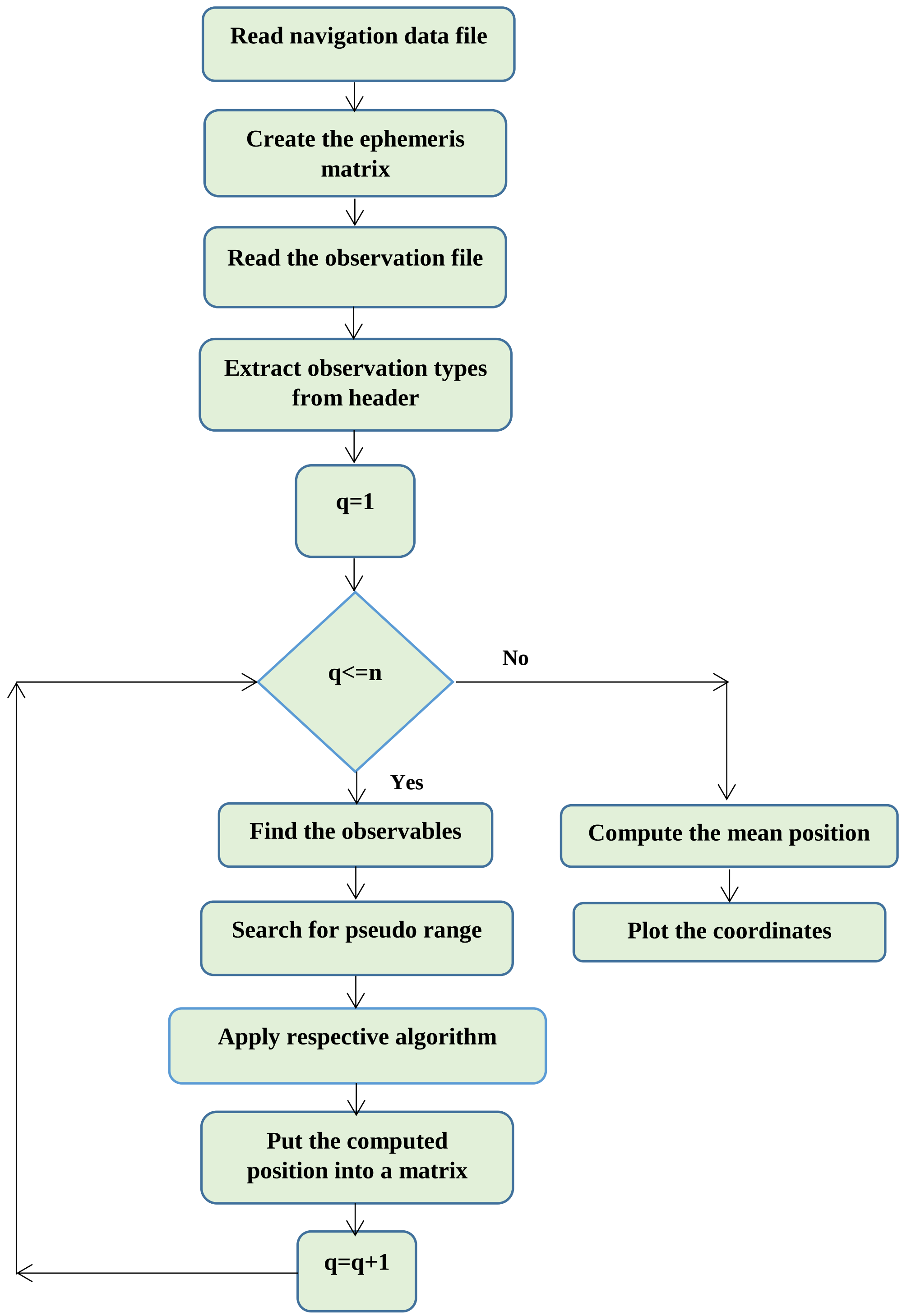
$\boldsymbol{R_{k+1}} = \boldsymbol{updated\ differential\ pseudo\ range.}$

The above set of equations forms the mathematical description of RLS algorithm.

CHAPTER 5

IMPLEMENTATION OF ALGORITHMS

5.1 THE PRELIMINARY PROCEDURE



The flowchart shown above goes like this:

- The program starts with the process of reading navigation data file from which ephemeris matrix is constructed and observation types are extracted from the header of observation data file.
- For each epoch, find the number of satellites in the view of the receiver. Now, extract pseudo range and apply one of the algorithms to compute the position and place it into a matrix.
- When the required numbers of computations for 'n' epochs have been performed, calculate the mean position and plot the coordinates.

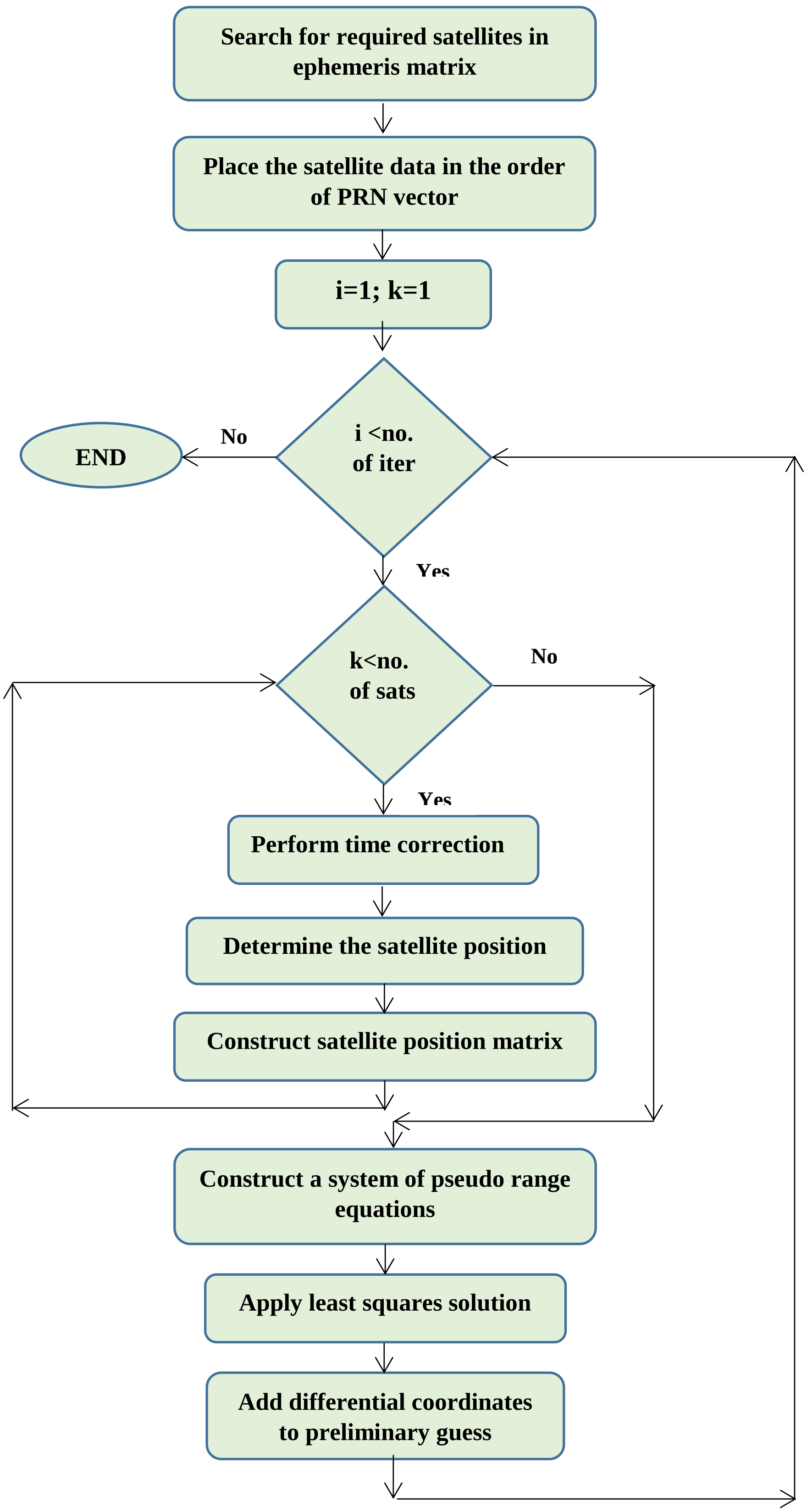
5.2 LEAST MEAN SQUARES ALGORITHM

As mentioned in the preliminary procedure, when least mean squares algorithm is called, the procedure followed is as shown below:

- Required satellites PRN codes are searched for in the ephemeris matrix with the help of satellite PRN vector from the observation file.
- For all the satellites, perform time corrections and determine their positions to construct a satellite position matrix.
- Using above computed satellite positions and pseudo ranges, construct a linearized system of equations about a neighborhood point and solve for differential coordinates using least squares method.
- Add the differential coordinates to the coordinates of neighborhood point iteratively (for six times) which converges to the true solution.

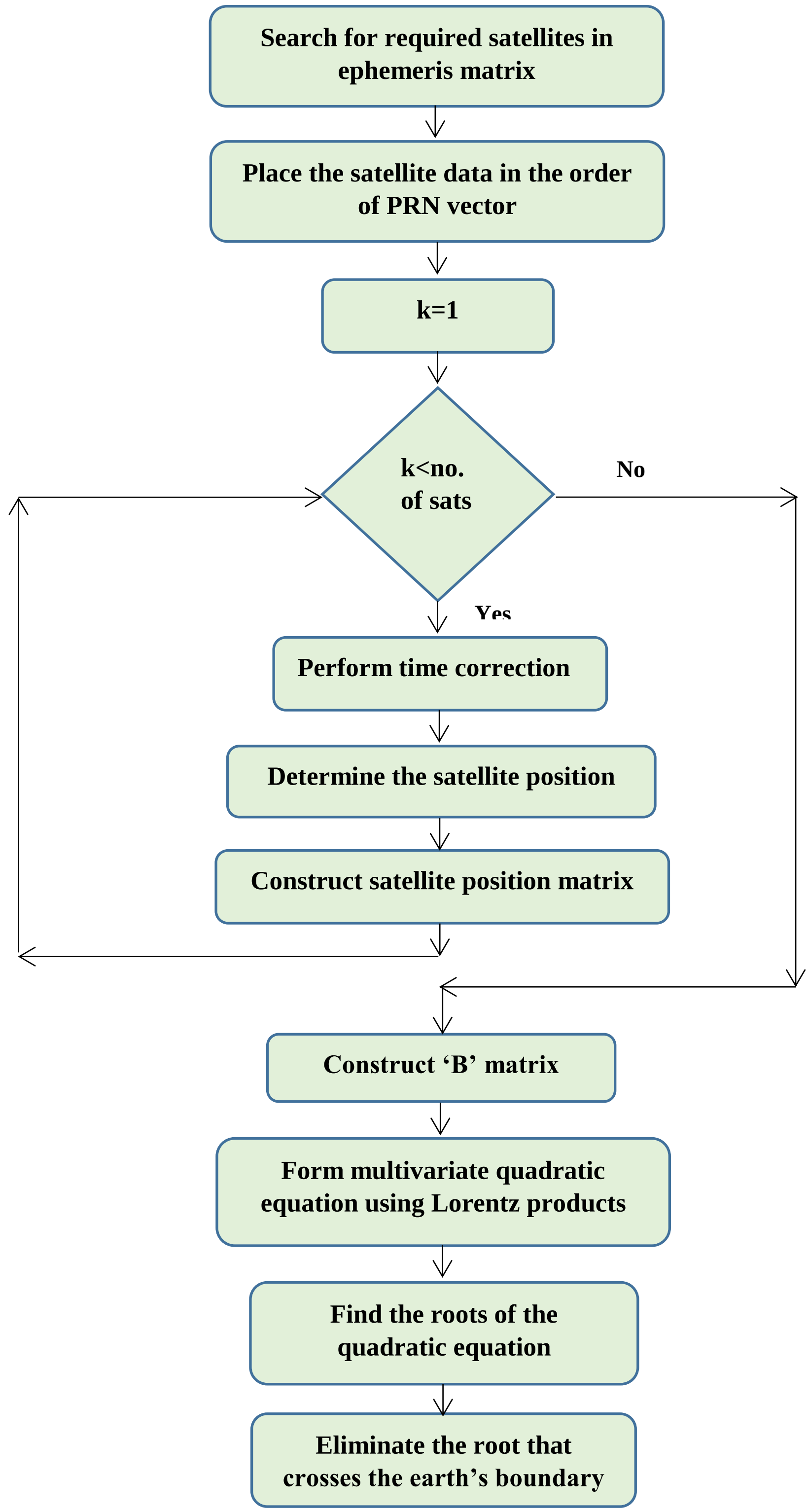
The above procedure for the least mean squares algorithm is depicted in the form of a flow chart as shown below:

FLOWCHART:



Search for required satellites in ephemeris matrix
Place the satellite data in the order of PRN vector
i=1; k=1
i <no. of iter
No
END
Yes
k<no. of sats
No
Yes
Perform time correction
Determine the satellite position
Construct satellite position matrix
Construct a system of pseudo range equations
Apply least squares solution
Add differential coordinates to preliminary guess

5.3 BANCROFT ALGORITHM



As mentioned in the preliminary procedure, when Bancroft algorithm is called, the procedure followed is as shown below:

- Required satellites PRN codes are searched for in the ephemeris matrix with the help of satellite PRN vector from the observation file.
- For all the satellites, perform time corrections and determine their positions to construct a satellite position matrix.
- Using the computed satellite positions and pseudo ranges, construct ‘B’ matrix to form multivariate quadratic equations in the form of Lorentz products.
- A quadratic equation gives two roots as its solution out of which one root has to be eliminated considering that it crosses the earth’s boundary.
- This root obtained after elimination gives the user position.

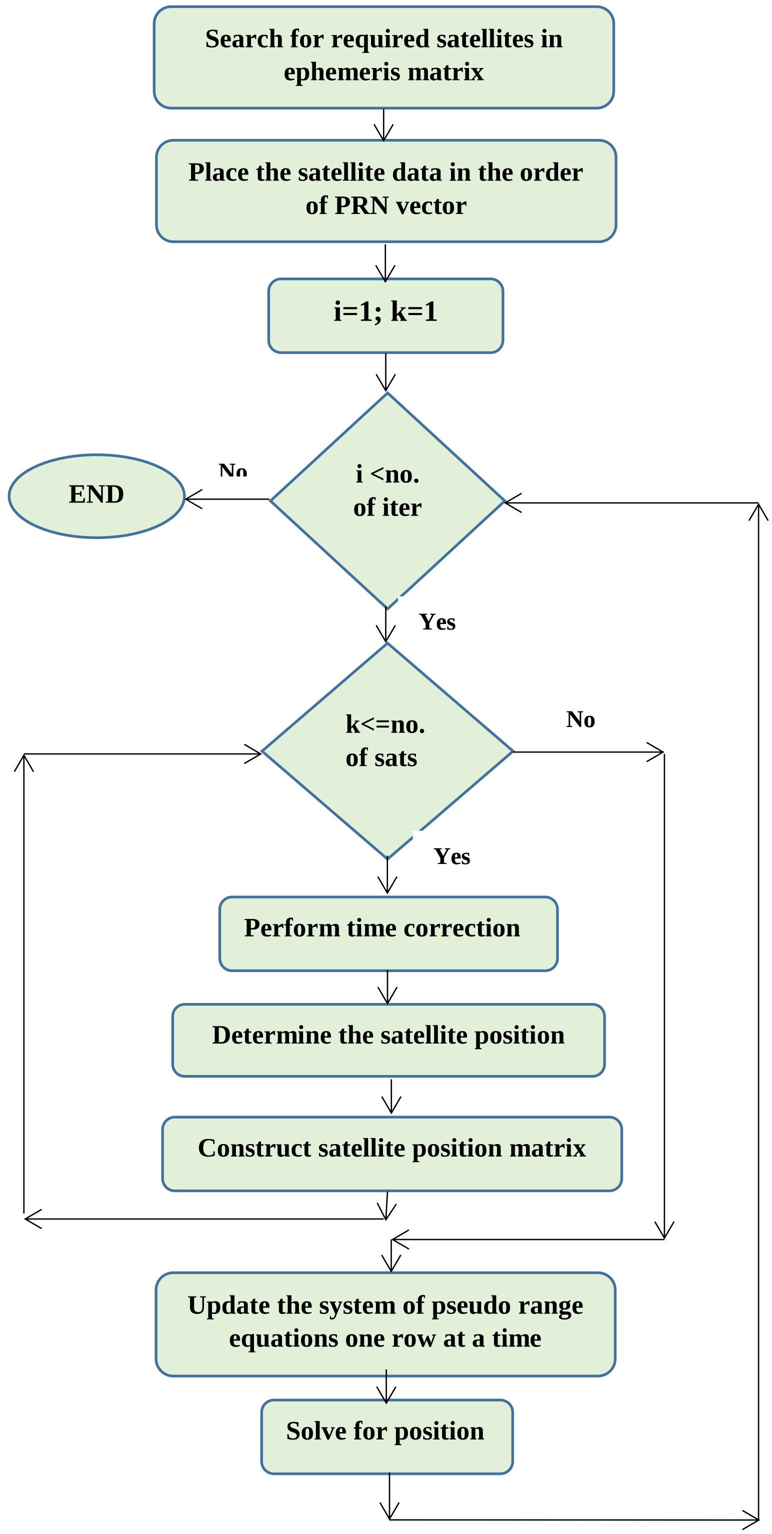
5.4 RECURSIVE LEAST SQUARES ALGORITHM

As mentioned in the preliminary procedure, when recursive least squares algorithm is called, the procedure followed is as shown below:

- Required satellites PRN codes are searched for in the ephemeris matrix with the help of satellite PRN vector from the observation file.
- For all the satellites, perform time corrections and determine their positions to construct a satellite position matrix.
- Using above computed satellite positions and pseudo ranges, construct a linearized system of equations which are updated one row at a time.
- Solve for user position iteratively till the order of coefficient matrix is completed.

The above procedure for the least mean squares algorithm is depicted in the form of a flow chart as shown below:

FLOWCHART:


Search for required satellites in ephemeris matrix
Place the satellite data in the order of PRN vector
i=1; k=1
i <no. of iter
No
END
Yes
k<=no. of sats
No
Yes
Perform time correction
Determine the satellite position
Construct satellite position matrix
Update the system of pseudo range equations one row at a time
Solve for position

CHAPTER 6

RESULTS

6.1 RESULTS

The algorithms are implemented in MATLAB. The program takes the RINEX navigation file and observation file as inputs, extracts the satellite ephemerides from navigation file to find its position in the orbit and to know the GPS time information and pseudo range information from observation file.

The results are simulated for the IRNSS constellation using the data received from the receiver placed at NCRC laboratory, ECE block, CBIT, Hyderabad.

The data used is taken from RINEX file which is dated 29-8-2016. The navigation data is taken during 20:00 hrs. UTC and the observation data is also taken with the starting epoch at 20:00 hrs. UTC.

6.1.1 SATELLITE COORDINATES

The ECEF coordinates of the satellite in IRNSS constellation, which are used for this project simulation, are shown in table 6.1:

Table 6.1: Position of satellites

IRNSS PRN	X-Coordinate (m)	Y-Coordinate (m)	Z-Coordinate (m)
I 03	4902369.99380994	41821410.1476929	2545921.10793719
I 04	-13634398.5118206	34037273.7361468	20819619.0146731
I 05	-14058384.7187245	34342179.0001880	-20025991.9757633
I 06	35663080.2337934	22525020.8972129	2270484.42725484
I 07	-26830729.8551970	32431920.0106051	2435691.10163341

6.1.2 LEAST MEAN SQUARE ALGORITHM RESULT

The iterative linearization of finding least squares solution is performed for 6 times after which the solution converges to the estimated location.

The mean is taken from 42 epochs of position estimate. The mean ECEF coordinates of the receiver are found to be

X: 1232719.370 m

Y: 5962801.743 m

Z: 1894377.972 m

The data of ECEF position coordinates computed using LMS algorithm for 42 epochs of Observation data is shown in table 6.2.

Table 6.2: Estimates of position coordinates

epoch	X pos (m)	Y pos(m)	Z pos (m)
1	1232706.559	5962795.360	1894329.860
2	1232707.165	5962795.823	1894332.071
3	1232708.044	5962796.027	1894334.660
4	1232708.457	5962796.220	1894336.776
5	1232708.985	5962796.344	1894339.174
6	1232709.602	5962796.452	1894341.459
7	1232710.434	5962796.810	1894344.197
8	1232711.074	5962797.557	1894346.462
9	1232711.623	5962797.748	1894348.552
10	1232712.282	5962797.558	1894351.355
11	1232712.746	5962798.205	1894353.351
12	1232713.089	5962797.947	1894355.306
13	1232713.781	5962798.558	1894357.714
14	1232714.598	5962798.989	1894359.849
15	1232715.278	5962799.499	1894362.826

16	1232715.928	5962799.885	1894365.087
17	1232716.601	5962800.301	1894367.147
18	1232717.126	5962800.368	1894369.426
19	1232717.743	5962800.941	1894372.097
20	1232718.529	5962801.339	1894374.799
21	1232719.109	5962801.486	1894377.409
22	1232719.582	5962801.936	1894379.200
23	1232720.339	5962802.083	1894381.457
24	1232721.060	5962802.768	1894383.902
25	1232721.522	5962802.592	1894386.315
26	1232722.133	5962802.858	1894388.291
27	1232722.819	5962803.908	1894390.688
28	1232723.348	5962804.246	1894393.108
29	1232723.923	5962804.378	1894395.452
30	1232724.630	5962804.334	1894398.017
31	1232725.191	5962804.571	1894400.548
32	1232725.877	5962804.718	1894402.751
33	1232726.576	5962805.503	1894405.199
34	1232727.230	5962805.991	1894407.561
35	1232727.998	5962806.846	1894409.647
36	1232728.605	5962806.588	1894411.987
37	1232729.069	5962806.585	1894414.393
38	1232729.622	5962807.254	1894417.069
39	1232730.308	5962807.401	1894419.005
40	1232731.071	5962807.912	1894421.200
41	1232731.573	5962808.487	1894423.517
42	1232732.310	5962808.813	1894425.924

Table 6.3 shows the comparison between latitude and longitude of position estimate with true position

Table 6.3: Comparison of position with error

	Estimated position	Expected Position	error
Latitude	17.3910 N	17.39098 N	-0.00002
Longitude	78.3196 E	78.31835 E	-0.00125

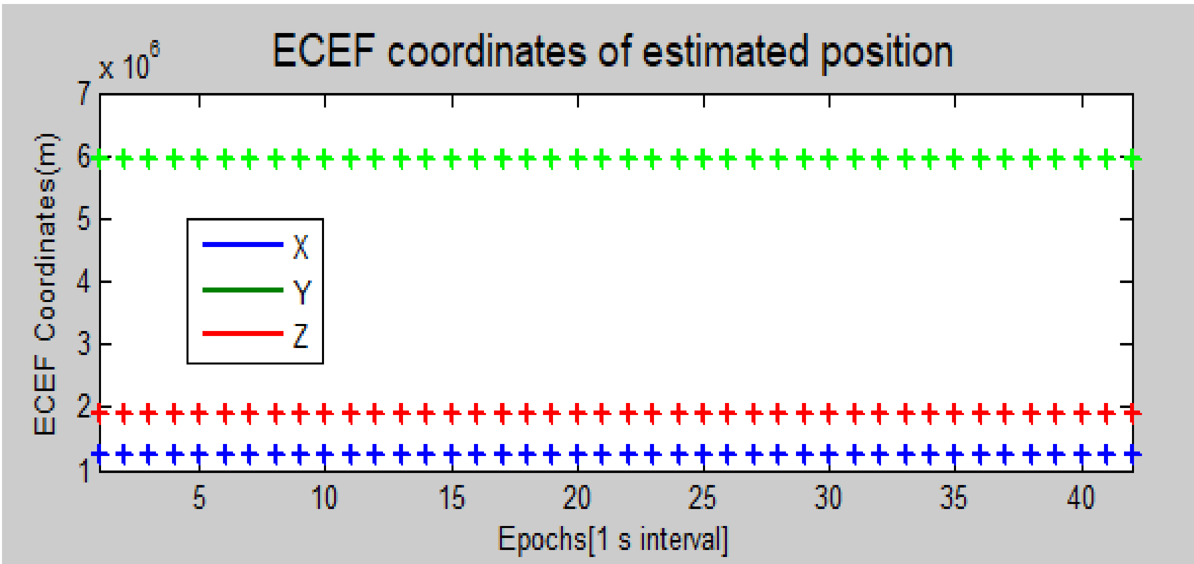


Figure 6.1: ECEF coordinates of the receiver computed using LMS algorithm

The figure 6.1 shows the plot of ECEF X, Y and Z coordinates with respect to 42 epochs, each epoch is spaced with an interval of 1 second.

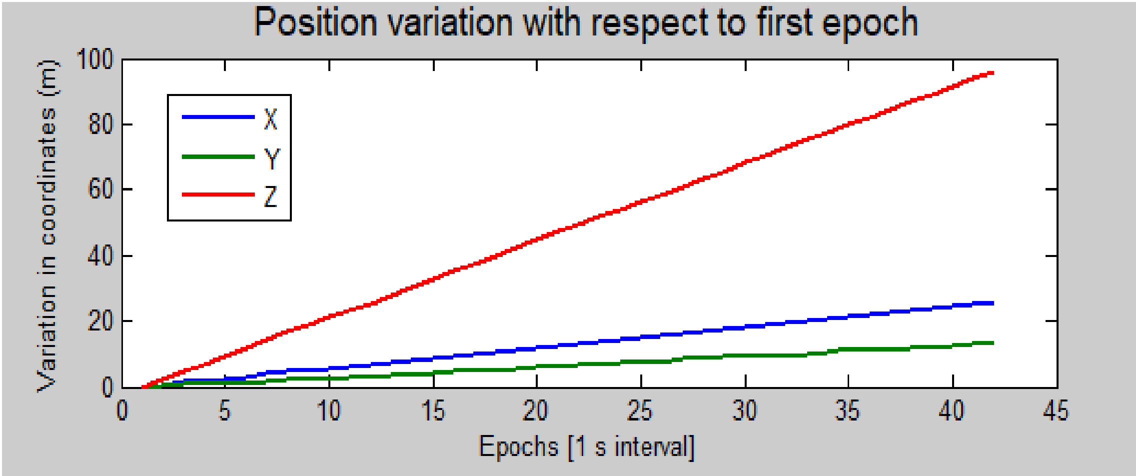


Figure 6.2: Variation of each coordinate with respect to first epoch

Figure 6.2 shows the variation of each coordinate value with respect to first epoch.

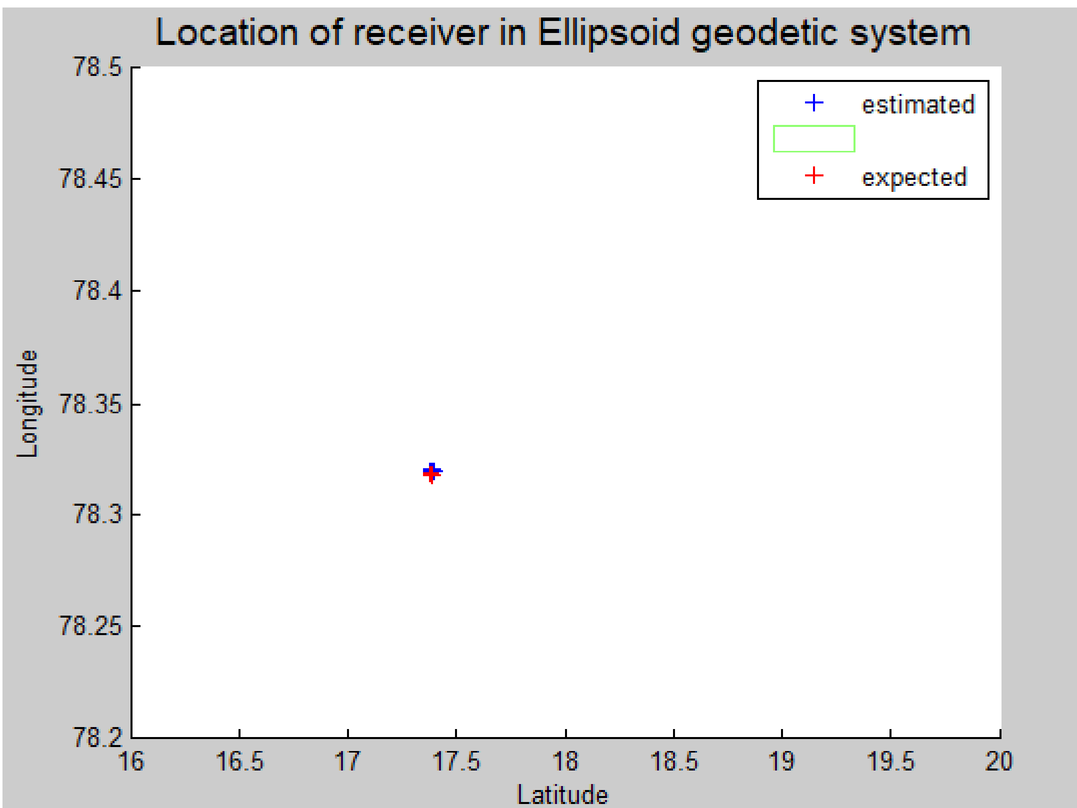


Figure 6.3: Location of receiver in ellipsoid coordinates

Figure 6.3 shows the geodetic latitude and longitude coordinates of receiver computed using LMS algorithm.

Function Name	Calls	Total Time	Self Time*	Total Time Plot (dark band = self time)
recpo_IsIRNSS	42	0.534 s	0.180 s	

Figure 6.4: Time of Execution of the base function of LMS algorithm

Figure 6.4 depicts the self-time and total time taken by the base function of LMS algorithm for all 42 function calls (one call per epoch).

6.1.3 BANCROFT ALGORITHM RESULT

The Bancroft algorithm doesn't involve the iterative linearization. It is a single time quadratic equation solving algorithm. Since it doesn't involve iterations, the execution time is fast compared to other two algorithms, but the accuracy of Bancroft is less compared to least mean squares algorithm.

The data of ECEF position coordinates computed using Bancroft algorithm for 42 epochs of Observation data is shown in table 6.4:

Table 6.4: Estimates of position coordinates

Epoch	X pos (m)	Y pos(m)	Z pos (m)
1	1232576.664	5962761.622	1894106.912
2	1232578.565	5962757.924	1894109.173
3	1232580.745	5962753.973	1894111.807
4	1232582.453	5962750.008	1894113.965
5	1232584.276	5962745.974	1894116.406
6	1232586.186	5962741.916	1894118.736
7	1232588.313	5962738.109	1894121.526
8	1232590.250	5962734.712	1894123.836
9	1232592.094	5962730.755	1894125.960
10	1232594.044	5962726.388	1894128.811
11	1232595.803	5962722.903	1894130.840
12	1232597.441	5962718.501	1894132.815

13	1232599.432	5962714.984	1894135.257
14	1232601.552	5962711.293	1894137.416
15	1232603.521	5962707.640	1894140.445
16	1232605.465	5962703.884	1894142.739
17	1232607.440	5962700.187	1894144.816
18	1232609.253	5962696.097	1894147.128
19	1232611.163	5962692.533	1894149.835
20	1232613.245	5962688.796	1894152.569
21	1232615.105	5962684.757	1894155.229
22	1232616.877	5962681.108	1894157.026
23	1232618.935	5962677.139	1894159.294
24	1232620.958	5962673.730	1894161.752
25	1232622.703	5962669.383	1894164.197
26	1232624.607	5962665.525	1894166.187
27	1232626.584	5962662.454	1894168.614
28	1232628.407	5962658.675	1894171.049
29	1232630.276	5962654.688	1894173.403
30	1232632.278	5962650.524	1894175.979
31	1232634.126	5962646.620	1894178.533
32	1232636.103	5962642.640	1894180.748
33	1232638.093	5962639.310	1894183.214
34	1232640.044	5962635.701	1894185.581
35	1232642.114	5962632.481	1894187.664
36	1232644.005	5962628.073	1894190.018
37	1232645.757	5962623.950	1894192.428
38	1232647.599	5962620.511	1894195.120
39	1232649.578	5962616.550	1894197.051
40	1232651.635	5962612.958	1894199.247
41	1232653.430	5962609.447	1894201.562
42	1232655.460	5962605.671	1894203.968

The table 6.5 shows the comparison between latitude and longitude of the nearest position estimate with true position:

Table 6.5: Comparison of position with error

	Estimated position	Expected Position	Error
Latitude	17.3878 N	17.39098 N	0.00318
Longitude	78.3193 E	78.31835 E	-0.00095

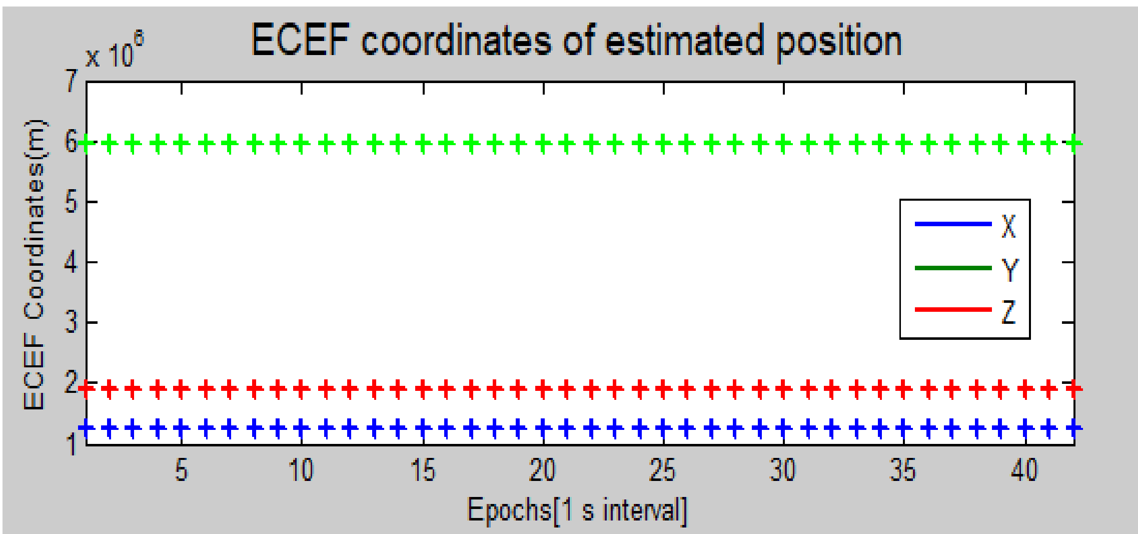


Figure 6.5: ECEF coordinates of the receiver computed using Bancroft algorithm

The figure 6.5 shows the plot of ECEF X, Y and Z coordinates with respect to 42 epochs each epoch is spaced with an interval of 1 second.

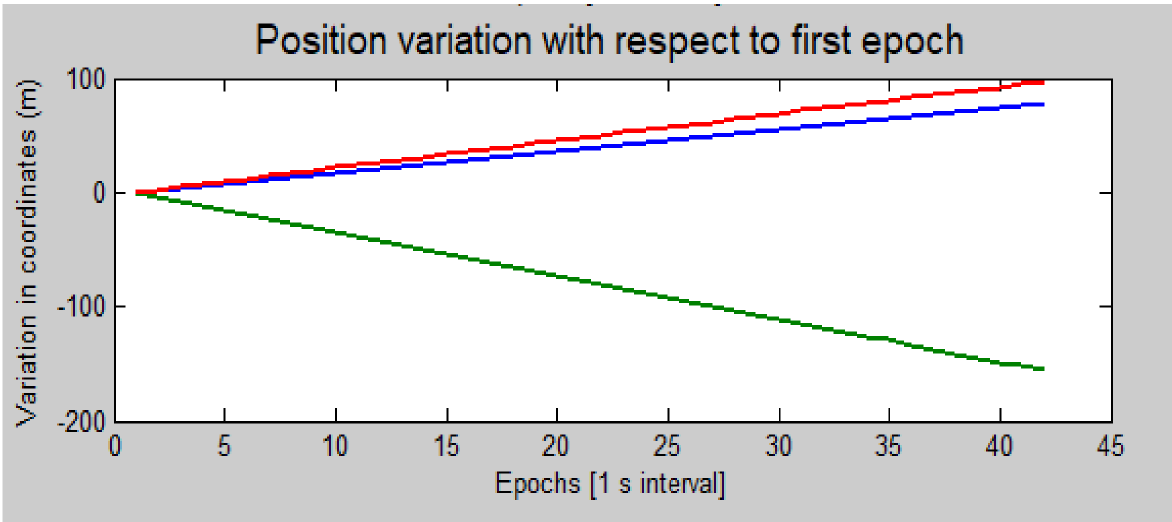


Figure 6.6: Variation of coordinate values at each epoch with respect to first epoch

Figure 6.6 shows the variation of each coordinate value with respect to first epoch.

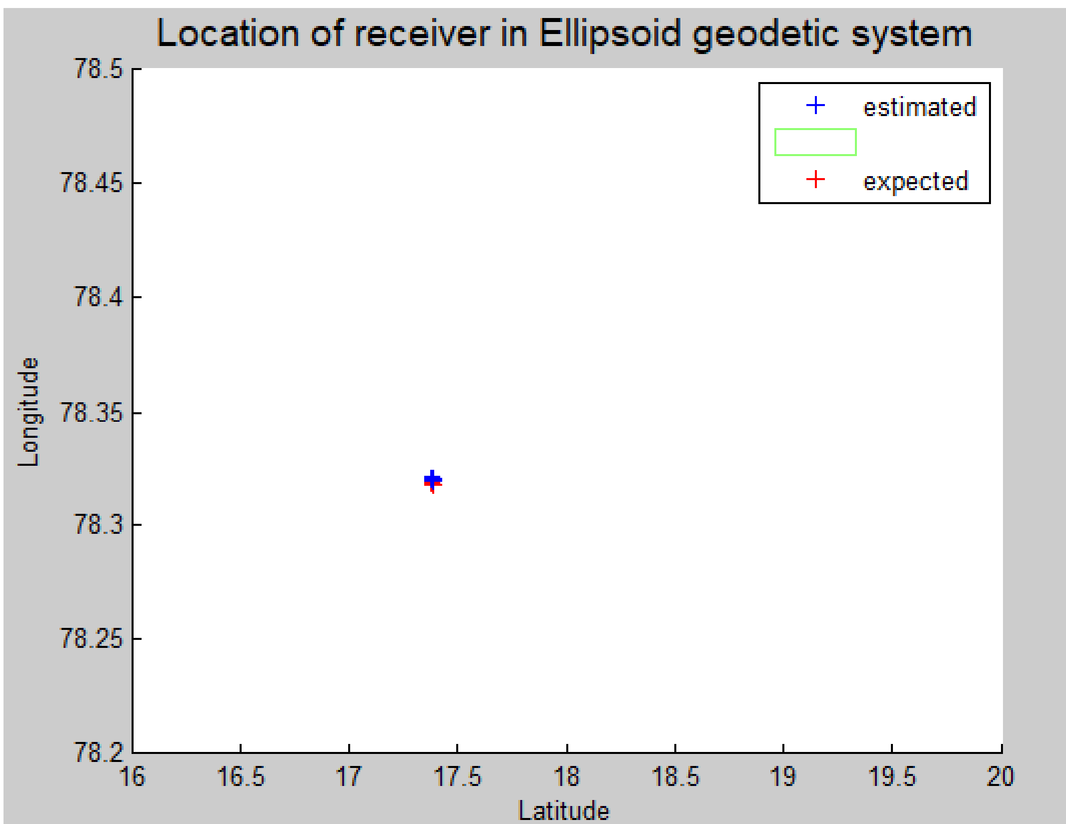


Figure 6.7: Location of receiver in ellipsoid coordinates

Figure 6.7 shows the geodetic latitude and longitude coordinates of receiver computed using Bancroft algorithm.

Function Name	Calls	Total Time	Self Time*	Total Time Plot (dark band = self time)
recpo_bancIRNSS	42	0.176 s	0.046 s	

Figure 6.8: Time of Execution of the base function of Bancroft algorithm

Figure 6.8 depicts the self-time and total time taken by the base function of Bancroft algorithm for all 42 function calls (one call per epoch).

6.1.4 RECURSIVE LEAST SQUARE ALGORITHM RESULT

The RLS algorithm includes iterative linearization and incremental update of the coefficient matrix A in the system of linearized pseudo range equations. It takes more time compared to both LMS and Bancroft algorithms. Since it is an algorithm with incremental updates, the variance in the position estimate is found to be large, but the circle error of probability radius is small with center as the expected position. The data of ECEF position coordinates computed using RLS algorithm for 42 epochs of observation data is shown in table 6.6.

Table 6.6: Estimates of position coordinates

epoch	X pos (m)	Y pos(m)	Z pos (m)
1	1277768.523	6653219.349	1568420.519
2	1234297.670	5969635.803	1894360.012
3	1225349.854	5927256.379	1901624.356
4	1208006.001	5854136.664	1884875.089
5	1242871.823	5921917.889	1895680.594
6	1218002.910	5878045.652	1890076.633
7	1226811.349	5933406.297	1895623.747
8	1231560.432	5958386.387	1894360.447
9	1230693.950	5952680.206	1889519.360
10	1239540.431	5983612.389	1872759.359

11	1226595.608	5897837.590	1912388.191
12	1232611.401	5954436.801	1893985.509
13	1209670.034	5860006.049	1893365.405
14	1222939.793	5936315.978	1893802.601
15	1241244.865	5997355.995	1900466.304
16	1205357.714	5787408.587	1891584.105
17	1228318.094	5948634.611	1890298.604
18	1231249.667	5948444.323	1891791.802
19	1236189.838	5977001.025	1893303.045
20	1226509.485	5937035.568	1895719.773
21	1359719.482	6560767.871	1905928.086
22	1229848.101	5951004.863	1896530.610
23	1230305.487	5966991.027	1868741.774
24	1221108.187	5927469.269	1916663.081
25	1212231.320	5903998.837	1889213.103
26	1227891.056	5945396.795	1897248.923
27	1221640.848	5899113.825	1892992.956
28	1235921.710	5966244.512	1890304.697
29	1233160.711	5956272.796	1895773.556
30	1202380.840	5804231.482	1868779.779
31	1228827.496	5946605.145	1891175.063
32	1234325.013	5962339.701	1893532.046
33	1226955.916	5933018.749	1893427.052
34	1230800.827	5952202.404	1892055.867
35	1228421.620	5931783.381	1901270.437
36	1192516.646	5865424.678	1804571.628
37	1229921.528	5951373.348	1893725.984
38	1230447.053	5956180.957	1890742.481
39	1189773.750	5607531.540	1857378.551
40	1232761.312	5961828.756	1892282.499

41	1224368.878	5937368.427	1898402.508
42	1233305.293	5979604.395	1888034.825

The table 6.7 shows the comparison between latitude and longitude of the nearest position estimate with expected position:

Table 6.7: Comparison of position with error

	Estimated position	Expected position	Error
Latitude	17.3837 N	17.39098 N	-0.00728
Longitude	78.3227 E	78.31835 E	-0.00435

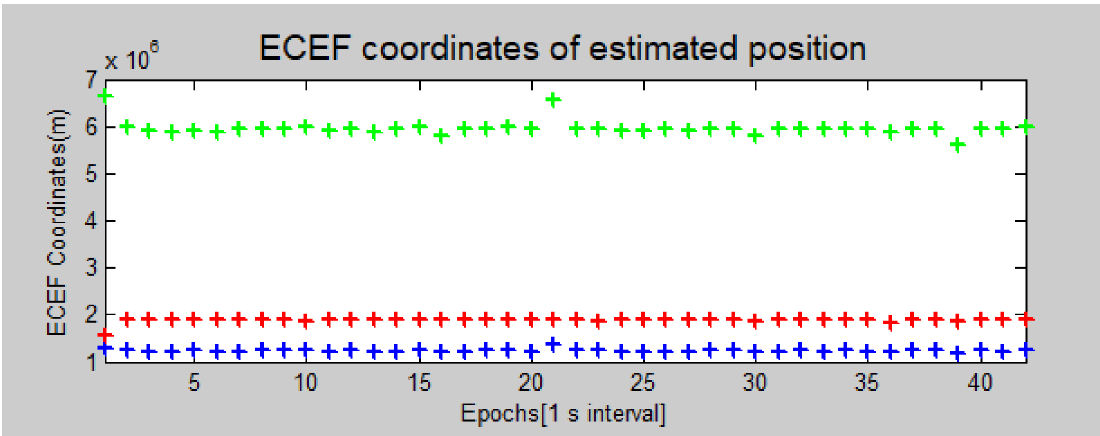


Figure 6.9: ECEF coordinates of the receiver computed using RLS algorithm

The figure 6.9 shows the plot of ECEF X, Y and Z coordinates with respect to 42 epochs each epoch is spaced with an interval of 1 second.

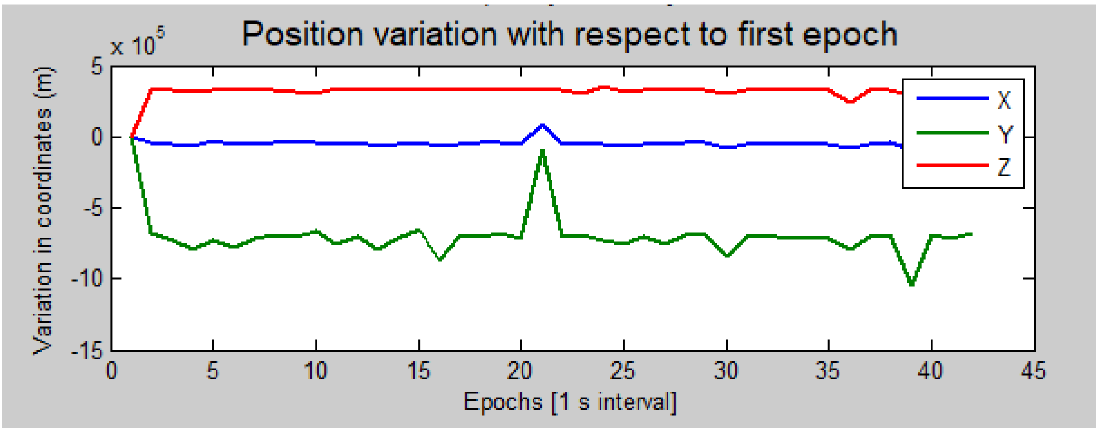


Figure 6.10: Variation of coordinate values at each epoch with respect to first epoch

Figure 6.10 shows the variation of each coordinate value with respect to first epoch.

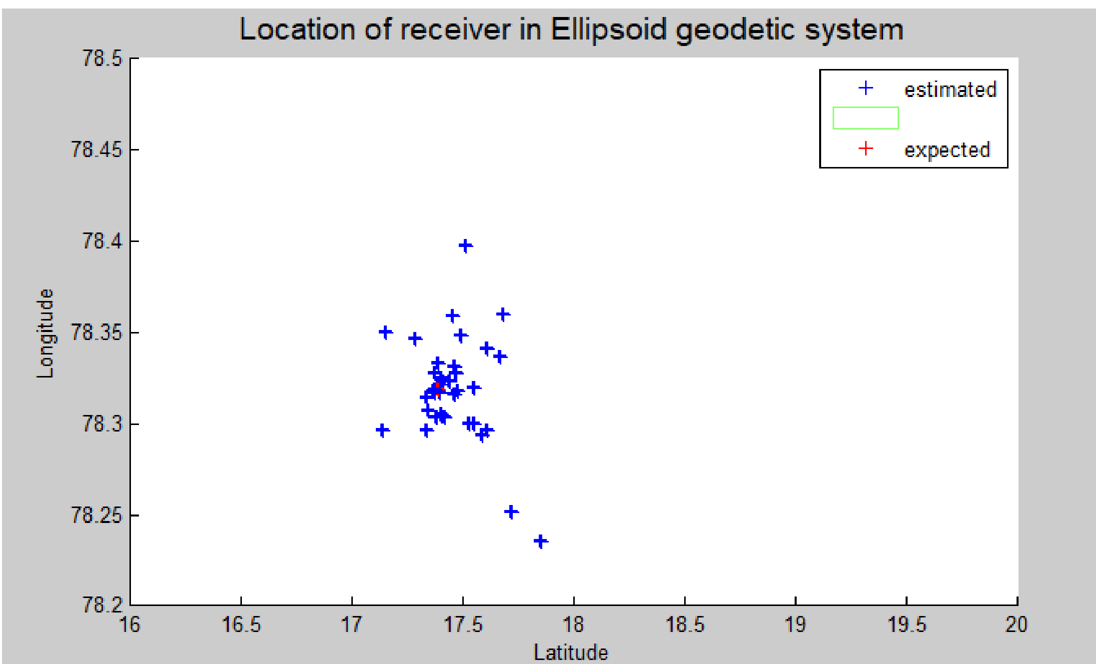


Figure 6.11: Location of receiver in ellipsoid coordinates

Figure 6.11 shows the geodetic latitude and longitude coordinates of receiver computed using RLS algorithm shown in blue and the true position is shown in red.

Function Name	Calls	Total Time	Self Time*	Total Time Plot (dark band = self time)
recpo_rlsIRNSS	42	2.422 s	0.270 s	

Figure 6.12: Time of Execution of the base function of RLS algorithm

Figure 6.12 depicts the self-time and total time taken by the base function of RLS algorithm for all 42 function calls (one call per epoch).

6.2 COMPARATIVE ANALYSIS OF ALGORITHMS

The comparative analysis of least mean squares, Bancroft and recursive least squares algorithms is shown below in table 6.8 as follows:

Table 6.8 Comparative analysis of Algorithms

Property	**Least Mean Squares**	**Bancroft**	**Recursive least squares**
Type	Iterative linearization and iterative addition of differential positions	Non-iterative quadratic solving algorithm	Iterative linearization; Incremental coefficient matrix update and iterative addition of differential positions
Accuracy	Good	Less than LMS	Less compared to Bancroft and LMS
Complexity	Moderate	Low (Direct solution)	Higher than LMS and Bancroft
Execution speed*	Slower than Bancroft	Fast	Slow compared to both LMS and Bancroft

- Type defines the approach to the position solution from pseudo range equations.
- Accuracy tells about how proximal the solution is to the true position of the Receiver.
- Complexity is defined based on the number of iterations performed and child function calls.
- Execution speed is quantized by using the run-in-time tool in MATLAB which profiles the time taken by each function another thing to consider is that the execution time depends on the processor used for executing the program(*Using the processor Intel(R) Core(TM) i3-3217U CPU @ 1.80GHz).

CHAPTER 7

CONCLUSIONS AND FUTURE SCOPE

7.1 CONCLUSIONS

In this project, the implementation and analysis of three GNSS receiver positioning algorithms is performed by plotting the coordinates and its variation using MATLAB program simulation.

Based on the comparison of accuracy and execution time, strengths and weaknesses of these algorithms are identified as follows:

1. The LMS algorithm which is an iterative algorithm performs the least squares solution and takes the mean of position estimates. The iterative linearization process is found to converge to the solution in 6 iterations. Out of the three algorithms, this algorithm gives a good accuracy of position estimate, though its complexity and execution speed is intermediate between Bancroft and RLS algorithm. This algorithm has its applications in areas requiring high accuracy without critical requirement on execution time.
2. The Bancroft algorithm's execution speed is high, since the solution is similar to finding the roots of a multivariate quadratic equation and there is no iterative linearization, but when it comes to accuracy it is found to be less than LMS algorithm. This algorithm fits for applications requiring stringent time constraint but with a good amount of approximation in position solution.
3. The RLS algorithm is slower and a bit less accurate than LMS and Bancroft algorithm. But by augmenting the RLS algorithm with weighted update using the covariance matrices, the accuracy improves substantially. The RLS algorithm can also be augmented with Kalman's update algorithm which gives accuracy greater than LMS.

7.2 FUTURE SCOPE

This project is preliminary and several extensions are possible. It can be extended to other algorithms like weighted least squares, Kalman update algorithm and genetic search algorithms. Other set of possible extension is by using carrier phase measurements along with pseudo range (code) measurements. This can also be replicated for dynamic receivers.

This project provides the comparative analysis for positioning algorithms, which can guide the intended users to choose the optimal algorithm based on application requirements.